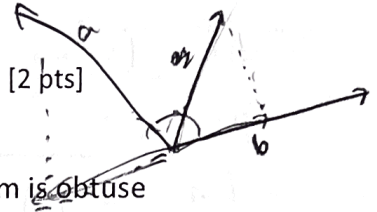


Questions 1 and 2 are Multiple Choice. Circle the best answer.

1. If the scalar projection $\text{proj}_b a$ is negative, what can be concluded about vectors a and b ? [2 pts]



a) The angle between them is acute

b) The angle between them is obtuse

c) $|a| > |b|$

d) $|a| < |b|$

e) None of these must be true



2. The set of parametric equations

$$x = 2 \cos(t) + 2$$

$$y = 3 \sin^2(t) - 5$$

is a.... (hint: eliminate the parameter) [2 pts]

a) circle

b) ellipse

c) ray

d) partial parabola

e) hyperbola



3. Lines l_1 and l_2 are parallel lines whose parametric equations are given. Which answer choice represents a parametric equation of the plane that contains the two lines? [2 pts]

$$l_1: \begin{cases} x = 1 + \frac{-1}{\sqrt{14}}d \\ y = -1 + \frac{2}{\sqrt{14}}d \\ z = 3 + \frac{3}{\sqrt{14}}d \end{cases}$$

$$l_2: \begin{cases} x = 5 + \frac{-1}{\sqrt{14}}d \\ y = 4 + \frac{2}{\sqrt{14}}d \\ z = -2 + \frac{3}{\sqrt{14}}d \end{cases}$$

a) $\langle x, y, z \rangle = \langle 1, -1, 3 \rangle + s\langle -1, 2, 3 \rangle + t\langle 5, 4, -2 \rangle$

b) $\langle x, y, z \rangle = \langle -1, 2, 3 \rangle + s\langle 1, -1, 3 \rangle + t\langle 5, 4, -2 \rangle$

c) $\langle x, y, z \rangle = \langle 1, -1, 3 \rangle + s\langle 4, 5, -5 \rangle + t\langle -1, 2, 3 \rangle$

d) $\langle x, y, z \rangle = \langle 1, -1, 3 \rangle + s\langle 4, 5, -5 \rangle + t\langle 5, 4, -2 \rangle$

e) none of the above

$$\langle 5, 4, -2 \rangle + d \langle \frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \rangle$$

$$\langle 1, -1, 3 \rangle + d \langle \frac{-1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \rangle$$

$$\langle 4, 5, -5 \rangle$$

4. Given vectors a , b , and c , where each vector is orthogonal to each other vector, and $\vec{a} \times \vec{b} = \vec{c}$.

Circle **ALL** the statements that are true (no need to "show work"): [3 pts]

$\times \vec{b} \times \vec{a} = \vec{c}$

$\times \vec{a} \times \vec{c} = \vec{b}$

$\vec{c} \times \vec{a} = \vec{b}$

$\vec{b} \times \vec{c} = \vec{a}$

$\times \vec{c} \times \vec{b} = \vec{a}$

$\vec{b} \times \vec{a} = -\vec{c}$

unit vectors

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- ★ 5. Given the points $A = (-1, 3, 2)$ and $B = (5, 2, 5)$, and point P which is on the line segment \overline{AB} and is twice as far from A as it is from B. Find the coordinates of P. [3 pts]

$$A(-1, 3, 2) \quad B(5, 2, 5)$$

$$\overline{AB} = (6, -1, 3)$$

$$P = (3, 2\frac{1}{3}, 4)$$

$$(4, -\frac{2}{3}, 2) \text{ away from A}$$

6. A vector equation of a line is $r = (3i - 2j) + t(i + 4j)$ where the parameter t is any real number.

- a) Write the equation of the line in rectangular form. [1 pt]

$$r = \langle 3, -2 \rangle + t \langle 1, 4 \rangle$$

$$x = 3 + t$$

$$y = -2 + 4t$$

$$y = 4x + b$$

$$-2 = 12 + b$$

$$b = -14$$

$$y = 4x - 14$$

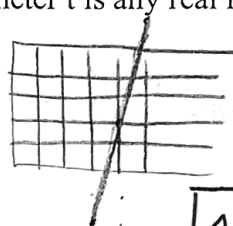
$$4x - y = 14$$

- b) Find the distance from the origin to the line. [2 pts]

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

$$\frac{14}{\sqrt{16 + 1}}$$

$$\text{distance} = \frac{14}{\sqrt{17}}$$



S, b

a, 2

- ★ 7. If $|a| = 2$, $|b| = 3$, and $a \cdot b = -6$,

- a) what is the angle θ between the vectors a and b ? [2 pts]

$$\cos \theta = \frac{u \cdot v}{|u||v|} = \frac{-6}{6} = -1$$

$$\theta = \pi$$

$$\langle 2, 0 \rangle \cdot \langle -3, 0 \rangle = -6 + 0 = -6$$

- b) What is the magnitude of the cross product $a \times b$? [2 pts]

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ -3 & 0 & 0 \end{vmatrix}$$

$$0\hat{i} - \hat{j} \begin{vmatrix} 2 & 0 \\ -3 & 0 \end{vmatrix} + \hat{k} \begin{vmatrix} 2 & 0 \\ -3 & 0 \end{vmatrix}$$

$$\text{magnitude} = 0$$

10

8. The unit vector $\mathbf{n} = \frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$ is perpendicular to a plane, K. The point $P = (2, 1, 2)$ is in the plane. Point Q is also in the plane, and the measure of angle MQP is 90 degrees. Point M $(-4, 4, 5)$ is not in the plane.

a) Find vector MP. [1 pt]

$$\langle 6, -3, -3 \rangle$$

b) Use projections to find the distance from the point M to the plane at Q. [2 pts]

$$\frac{1}{3}x - \frac{2}{3}y + \frac{2}{3}z - \frac{4}{3} = 0$$

$$\text{distance} = 2$$

$$\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}} = \frac{\left| -\frac{4}{3} - \frac{8}{3} + \frac{10}{3} - \frac{4}{3} \right|}{\sqrt{\frac{1}{9} + \frac{4}{9} + \frac{4}{9}}} = 2$$

formula is derived from projections

9. Find two values of k that make the vectors $\langle 11, 2, -7 \rangle$ and $\langle k, k^2, 3 \rangle$ orthogonal. [3 pts]

$$11x + 2x^2 - 21 = 0$$

$$(2x - 3)(x + 7) = 0$$

$$2x^2 + 11x - 21 = 0$$

$$x = -7 \quad x = \frac{3}{2}$$

$$2x^2 + 14x - 3x - 21 = 0$$

$$2x(x + 7) - 3(x + 7) = 0$$

$$k = -7 \text{ or } \frac{3}{2}$$

10. How many ways can you... [2 pts each]

a) rearrange the letters of CALCULUS?

$$\frac{8!}{2! \cdot 2! \cdot 2!}$$

b) arrange 9 students in a circle?

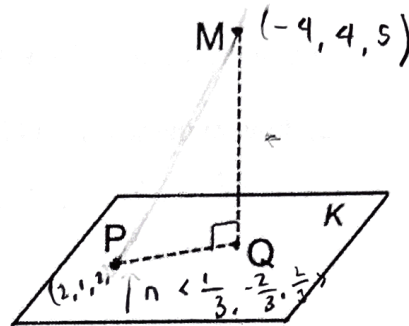
$$8! \text{ ways}$$

c) select a president, vice-president, and treasurer from a group of 14 student?

$$14 \cdot 13 \cdot 12$$

d) form a committee of 5 men and 7 women from a group of 20 men and 30 women?

$$\binom{20}{5} \binom{30}{7}$$



$$-7, 49$$

$$-77 + 49 - 21$$

$$\begin{array}{r} 98 \\ -77 \\ \hline 21 \\ + \frac{18}{4} \\ \hline \end{array} \quad \begin{array}{r} 49 \\ -21 \\ \hline \end{array}$$

-0

11. A bag contains 15 marbles, numbered 1-15. Marbles 1-10 are blue, and marbles 11-15 are red.

If you draw 3 marbles without replacement... [2 pts each]

a) how many ways could you draw all odd numbers?

1, 3, 5, 7, 9, 11, 13, 15

$$\binom{8}{3}$$

b) What is the probability that all 3 are blue?

$$\frac{\binom{10}{3}}{\binom{15}{3}}$$

~~★~~ c) What is the probability that the three are consecutive numbers?

$$\frac{\binom{13}{1}}{\binom{15}{3}}$$

12. A single, 6-sided die is rolled 4 times in a row, and the results are written in order, creating a 4-digit number. Find... [2 pts each]

a) P(the number is 4226) =

$$\frac{1}{6^4}$$

b) P(the number is a multiple of 5) =

$$\frac{\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}}{\frac{5}{6^4}}$$

c) P(the number has only odd digits) =

$$\frac{\frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6}}{\frac{3^4}{6^4}}$$

d) P(the number has no digit repeated) =

$$\frac{6 \cdot 5 \cdot 4 \cdot 3}{6^4}$$

$$\frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6}$$

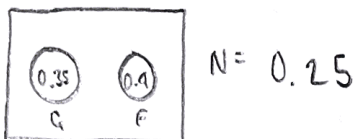
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Questions 13-15 refer to a certain population where:

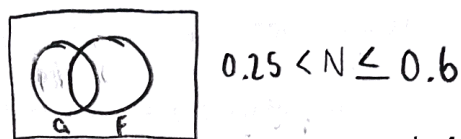
- the probability of being a "gamer" is 35% $\rightarrow P(G) = 0.35$
- the probability of being "physically fit" is 40% $\rightarrow P(F) = 0.40$
- Let $N = P(G' \cap F')$.

13. What are ALL the possible values of N if... [2 pts each]

a) G and F are mutually exclusive?



b) G and F are NOT mutually exclusive?



14. If 20 people are randomly selected from the population, what is the probability that exactly 4 of them will be "gamers"? [2 pts]

$$\binom{20}{4} (0.35)^4 (0.65)^{16}$$

15. If 20 people are randomly selected from the population, how many of them would you expect to be "physically fit"? [2 pts]

$$20 \cdot 0.4 = 8 \text{ people}$$

16. Jar A contains 4 blue and 2 red marbles. Jar B contains 3 blue and 3 red marbles. You choose one jar at random, and then chose a random marble from the jar. Find... [2 pts each]

a) $P(\text{blue marble} \mid \text{Jar A}) =$

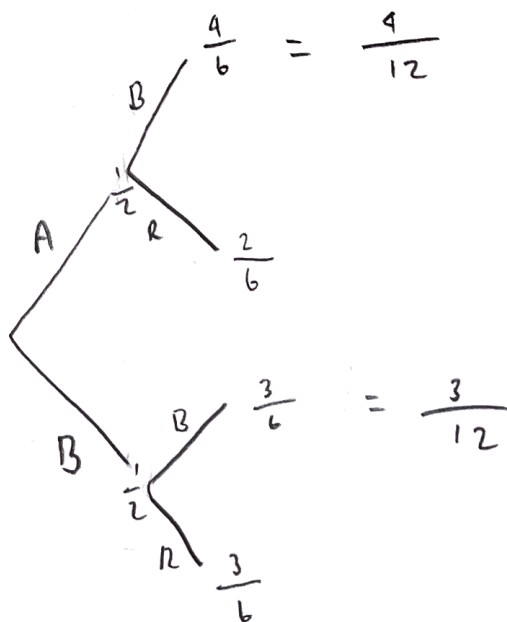
$$\frac{4}{6}$$

b) $P(\text{Jar A} \mid \text{blue marble}) =$

$$\frac{4}{7}$$

c) $P(\text{Jar A or blue marble}) =$

$$\frac{3}{4}$$



$$\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$$

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