Analysis Honors – Hahn/Tantod
Unit 8: Limits Quiz 1
NO CALCULATORS [27 points]

27/27

Chris Lee is in the 'hood.
Period: 3

you can grade

2 4 8 16 32
3 9 27 81 243

1. Does the sequence converge or diverge? [1 pt each]

a. $1, 8, 27, 64, \ldots, n^3, \ldots$ diverge

b. $1, \frac{2}{3}, \frac{4}{9}, \frac{8}{27}, \ldots, \left(\frac{2}{3}\right)^n, \ldots$ converge

c. $a_n = \frac{3n-1}{n+1}$ converge

d. $a_n = 4n - 3$ diverge

e. $a_n = \frac{2-2n}{2n+1}$ converge

f. $a_1 = 1, a_n = a_{n-1} + 3\ for\ n \geq 2$ diverge

1, 4, 7, 10, 13, 16

For Questions #2-5, answer True or False for each statement. [1 pt each]

2. If a sequence is always increasing, there must be a lower bound. T

3. If a sequence is bounded above and below, it must have a limit F

4. If a sequence is always increasing, it can't have a limit. F

5. Showing $a_n < n$ would show that a_n converges . F

For Questions #6-8, find the limit of each sequence, or say "diverges" if the sequence diverges. [2 each]

6. $a_n = \frac{\cos^2 n}{6n}$

0

7. $b_n = \frac{n+1}{\sqrt{n}}$ $\frac{n^1}{n^{\frac{1}{2}}}$ $\frac{145}{12}$ 2, 3, 12

diverges

8. $c_n = \frac{3-4n^2}{2+5n^2}$ $\frac{100 \cdot 10}{100}$ $\frac{10{,}000}{100}$

$-\frac{4}{5}$

9. Given the sequence $d_n = \frac{2n^2}{3n^2+1}$...

a) The limit of the sequence is $\underline{\frac{2}{3}}$. [2]

b) **Prove** your limit from part (a) using a neighborhood of radius 1/30. Include a conclusion statement. [4]

$$\frac{19}{30} \leq \frac{2n^2}{3n^2+1} < \frac{21}{30}$$

$$\frac{19}{30} \leq \frac{2n^2}{3n^2+1}$$

$$\frac{19}{30}(3n^2+1) \leq 2n^2$$

$$\frac{19}{10}n^2 + \frac{19}{30} \leq 2n^2$$

$$\frac{19}{30} \leq \frac{1}{10}n^2$$

$$\frac{19}{3} \leq n^2$$

$$\boxed{\sqrt{\frac{19}{3}} \leq n}$$

$$\frac{2n^2}{3n^2+1} < \frac{21}{30}$$

$$2n^2 \leq \frac{21}{30}(3n^2+1)$$

$$2n^2 \leq \frac{21}{10}n^2 + \frac{21}{30}$$

$$-\frac{1}{10}n^2 < \frac{21}{30}$$

$$n^2 > -\frac{21}{3}$$

always true

let $M = \sqrt{\frac{19}{3}}$ for all $n \geq M$, $\left\{\frac{2n^2}{3n^2+1}\right\}$ will be within $\frac{1}{30}$ of $\frac{2}{3}$.

$$\therefore \lim_{n\to\infty} d_n = \frac{2}{3}$$

10. Prove that the sequence $t_n = \frac{2n+1}{3n+2}$ converges to a limit by showing that it is everywhere increasing or everywhere decreasing (choose one), and bounded above or bounded below (choose one). Include a conclusion statement. [5]

$$\frac{2n+1}{3n+2} \leq \frac{2n+3}{3n+5}$$

$$(2n+1)(3n+5) \leq (3n+2)(2n+3)$$

$$\cancel{6n^2} + \cancel{13n} + 5 \leq \cancel{6n^2} + \cancel{13n} + 6$$

$$5 \leq 6$$

always true so everywhere increasing

$$\frac{2n+1}{3n+2} \leq 10$$

$$2n+1 \leq 10(3n+2)$$

$$2n+1 \leq 30n+20$$

$$\frac{-19}{28} \leq \frac{28n}{28}$$

$$n \geq \frac{-19}{28}$$

← always true so bounded above

Because t_n is always increasing and is bounded above, it converges to a limit (least upper bound).