Analysis H – Hahn / Tantod
Unit 1: AtPS, Quiz 1
NO CALCULATORS

23/25 24

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Period: 6

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1. The following statements all refer to the odd-numbered triangle (shown on the right).
Write "true" or "false" for each statement.

			1			
		3		5		
		7	9	11		
	13	15	17	19		
	21	23	25	27	29	
31	33	35	37	39	41	
43	45	47	49	51	53	55

a) The sum of the first k odd numbers is k^2. $\boxed{\text{true}}$

$1+2k-1$, $\frac{2k(k)}{2} = k^2$, 1st+last $\frac{2n+2}{2}$

b) The sum of any two triangular numbers is a square number $\boxed{\text{true}}$ not necessarily

1 3 6 1+6 ≠ perfect square

nth Δ lar # = $\frac{n(n+1)}{2}$ $\frac{n(n+1)}{2} + \frac{k(k+1)}{2}$ $\frac{(n-1)n}{2} + \frac{n(n+1)}{2}$ $\frac{n(n-1+n+1)}{2}$ $\frac{2n^2}{2} = n^2$

c) The sum of the nth row of the odd-numbered triangle is always a cube number. $\boxed{\text{true}}$ n^3.

$\frac{2n^3}{2} = n^3$

d) The sum of the first k cube numbers is a square number. $\boxed{\text{true}}$

1st n rows sum to $\left(\frac{n(n+1)}{2}\right)^2$.

e) The difference between the 1st term of the (n + 4)th row and the 1st term of the nth row is a square number. $\boxed{\text{false}}$

1st term $n(n-1)+1$
$(n+4)(n+3)+1$
$n^2+7n+12+1 = n^2+7n+13$
$-n^2+n-1$, $\boxed{8n+12}$ not a perf. square

$(n+4)^2-(n+4)+1$
$n^2+8n+16-n-4+1$
$n^2+7n+13 - (n^2-n+1)$
$+8n+12$.

2. Find the sum of each expression.

a) 30 + 34 + 38 + 42 + … + 150 =

$\frac{n}{2}(a_1+a_n)$. d = 4

$\frac{(30+150)31}{2}$

$= \frac{180 \cdot 31}{2} = 31 \cdot 90$

$= \boxed{2790}$ ✓

30 34 38 42 … 150
32 36 40 44 … 152
÷4 8 9 10 11 … 38
38−8+1 = 31 terms

$\frac{120}{4} = 30+1 = 31$ terms.

b) The first 15 terms of the following series (just give an expression for the answer – you don't have to calculate the actual number by hand):

$\frac{2}{3} = \frac{18}{27} = \frac{54}{81} = \frac{36}{54}$

81 + 54 + 36 + 24 + …

$\boxed{15 \text{ terms}}$ $\frac{a(1-r^n)}{1-r}$ = $\frac{81(1-(\frac{2}{3})^{15})}{1-\frac{2}{3}}$

$r = \frac{2}{3}$, $a = 81$, $n = 15$

$\frac{81(1-(\frac{2}{3})^{15})}{\frac{1}{3}} = \frac{81(1-(\frac{2}{3})^{15})}{\frac{1}{3}} = \boxed{243(1-(\frac{2}{3})^{15})}$ ✓

$243(1-(\frac{2}{3})^{15})$

3. Simplify each as a single term, or single binomial coefficient.

a) $\binom{18}{0} + \binom{18}{2} + \binom{18}{4} + \binom{18}{6} + \cdots + \binom{18}{18} = 2^{18-1} = \boxed{2^{17}}$ ✓

sum of every other term in row n is 2^{n-1} ↳ not double adding #s in prev row.

$\binom{17}{0}$ $\binom{17}{1}$ $\binom{17}{2}$
$\binom{18}{0}$ $\binom{18}{1}$ $\binom{18}{2}$
not added so $\binom{17}{0}$ & $\binom{17}{1}$ "used" just once

b) $\binom{37}{37} = \binom{38}{38} = 1$ $\binom{37}{37} + \binom{38}{37} + \binom{39}{37} + \binom{40}{37} + \cdots + \binom{82}{37} =$

$\binom{38}{38} + \binom{38}{37} + \binom{39}{37} + \cdots + \binom{82}{37}$

$= \binom{39}{38} + \binom{39}{37} + \cdots + \binom{82}{37} = \cdots = \binom{82}{38} + \binom{82}{37} = \boxed{\binom{83}{38}}$

$\sum_{k=0}^{k=p} \binom{n+k}{n} = \binom{n+k+1}{n+1}$

c) $\binom{52}{7} + 3\binom{52}{8} + 3\binom{52}{9} + \binom{52}{10} =$

$\binom{52}{7} + \binom{52}{8} + 2\binom{52}{8} + 2\binom{52}{9} + \binom{52}{9} + \binom{52}{10}$.

$\binom{53}{8} + 2\binom{53}{9} + \binom{53}{10}$

$\binom{53}{8} + \binom{53}{9} + \binom{53}{9} + \binom{53}{10}$

$= \binom{54}{9} + \binom{54}{10} = \boxed{\binom{55}{10}}$ −1

$\binom{54}{10} + \binom{54}{12} = \binom{55}{13}$

−2

4. The first 5 rows of triangular pattern is shown below, where all terms are multiples of 5. For reference, the bolded "40" is the 2nd term of the 4th row, and is also the 3rd term of the 2nd column. The bolded "70" is the 4th term of the 5th row, and also the 2nd term of the 4th column.

5
10 15
20 25 30
35 **40** 45 50
55 60 65 **70** 75
80 85 90 95 100 105

$5+5(n-1)$ is nth term.

This is amazing, but you really didn't have to write all this! :) Better to be concise and ensure the question!

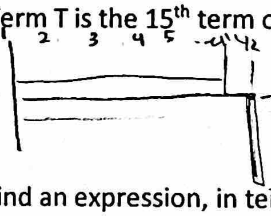
a) What is the 6th term of the 6th row?

6th term of 6th row is.

$1+2+3+4+5+6 = \frac{7(6)}{2} = \frac{42}{2} = 21\text{st term}$

$5+5(20) = \boxed{105}$

b) Term T is the 15th term of the 42nd column. In which row is term T?



42nd row was 15 col up row w/ term T.

42

15 inclusive.

$42+15-1 = 56$

$\boxed{56\text{th row}}$

c) Find an expression, in terms of *k*, for the 3rd term of the *k*th column. *This may be a challenge, so make sure you clearly label and organize your work, so that I can follow what you're doing and give partial credit!*

~~There are k column~~

For each row, 1 value more is added from previous, so row n-1 has ~~x~~-1 element, row n has (~~x~~-1)+1, n elements, etc since row 1 has 1 element, row n will have n elements,

Therefore, ~~the~~ for a value k, the kth column cannot exist unless the row # is ≥ k because all prev rows have k-1, k-2, ..., 0 columns because row # = # of elements. Therefore the 1st term of the kth column exists on the kth row.

to find 3rd term for kth column you simply go down 2 rows to find 3rd term. Therefore, the value exists on (k+2)nd row, kth ~~(k+2)nd~~ column.

1 | row k
2 | k+1
3 | k+2

~~In addition it is the kth term of the (k+2)nd row~~ because it is in

(cont on scratch paper.)

5. With Fibonacci numbers: $F_{400} = F_a F_{249} + F_b F_{248}$

Find *a* and *b*.

~~$F_{400} = F_3$~~.

$\boxed{\begin{array}{l} a = 152 \\ b = 151. \end{array}}$

$F_{a+n+1} = F_{a+1} F_{n+1} + F_a F_n$.

$F_{400} = F_{152} F_{249} + F_{151} F_{248}$.

248+1

400
249
151

3 9
400
248
152

$n = 248$

$a+n+1 = \cancel{248}\ 400$

$a + 249 = 400$

$a = 151.$

151, 152

3 9
400
249
151

151
249
400

$F_{a+n+1} = F_{a+1} F_{n+1} + F_a F_n$

$n = 248$

$= F_{a+1} F_{249} + F_a F_{248}$

$a + \cancel{248+1} = 400$

249 3 9
400
249
$a = 151$

$a+1 = 152$

$a = 151$

6. Express the following as a difference of two Fibonacci numbers. (hint: use telescoping!)

$F_{73} + F_{75} + F_{77} + F_{79} + \cdots + F_{853} =$

$= (F_{74} - F_{72}) + (F_{76} - F_{74}) + (F_{78} - F_{76}) + \cdots + (F_{852} - F_{850}) + (F_{854} - F_{852})$

$= \cancel{8}\ \boxed{F_{854} - F_{72}}$

2/0

$r = a + b\sin\theta$ / $\cos$

rose $r = a\sin n\theta$

lemniscate $r^2 = a^2 \sin 2\theta$ / $\cos$

dimple ~~limaçon~~ $\therefore 1 < \left|\frac{a}{b}\right| < 2$

inner loop. ~~dimple~~ $|b| > |a|$

convex $2 \le \left|\frac{a}{b}\right|$

cardioid $|b| = |a|$

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4 c). cont.

~~therefore,~~ using this info you find that we are looking for the.

prev full rows. $\frac{(k+1)(k+2)}{2}$ ↘ $\frac{(\#\text{ of rows})(\text{sum of 1st \& last row size})}{2}$

$=$ # of total terms till 1st term of k+2nd row (exclusive).

so the 3rd term of kth col is.

$\frac{(k+1)(k+2)}{2} + k$

total # of terms in prev (k+1) rows

kth term on k+2nd row

the ~~a~~ yth term in the triangle

$= 5 + 5(y-1)$ → terms go up by 5. from 5.

so

the 3rd term of kth column equals

$$5 + 5\left(\frac{(k+1)(k+2)}{2} + k - 1\right)$$

$$= \cancel{5} + \frac{5(k+1)(k+2)}{2} + 5k \cancel{-5}$$

$$= \boxed{\frac{5(k+1)(k+2)}{2} + 5k}$$

or.

$$= \frac{5(k^2+3k+2) + 5(2k)}{2} = \boxed{\frac{5(k^2+5k+2)}{2}}$$

Check.

$$5\left(k + \frac{(k+1)(k+1+1)}{2} - 1\right) + 5.$$

$$5k + \frac{5(k+1)(k+2)}{2} \cancel{-5} \cancel{+5}$$

$$\frac{10k + 5(k^2+3k+2)}{2} = \frac{5(k^2+5k+2)}{2}$$

3rd col.

$\frac{5(9+15+2)}{2} = \frac{5(26)}{2}$

$5(13)$

13
5
65 ✓

k^2+3k+2

check 2nd col

40

$\frac{5(4+10+2)}{2}$

$\frac{5(16)}{2} = 5(8) = 40$

k+2nd row

$$5\left(\frac{(k+1)(k+2)}{2} + k - 1\right) + 5$$