Analysis H 2021-2022 – Hahn / Tantod
Unit 3: Vectors, Quiz 2
No Calculators

28
30 points

Chris Lee will be vectorious!
Period: 3

1. Find the area of the triangle that has vectors $\overrightarrow{AB}$ $\langle -3,1,4\rangle$ and $\overrightarrow{AC}$ $\langle 6,-5,2\rangle$ as 2 of its sides. Since you don't have a calculator, don't try to simplify (just stop when you have a numerically equivalent answer). [4 pts]

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 4 \\ 6 & -5 & 2 \end{vmatrix} = \hat{i}\begin{vmatrix} 1 & 4 \\ -5 & 2 \end{vmatrix} - \hat{j}\begin{vmatrix} -3 & 4 \\ 6 & 2 \end{vmatrix} + \hat{k}\begin{vmatrix} -3 & 1 \\ 6 & -5 \end{vmatrix}$$

$$= 22i + 30j + 9k \qquad |\langle 22, 30, 9\rangle| =$$

$$\boxed{\frac{\sqrt{22^2+30^2+81}}{2} = \text{Area of } \triangle}$$

2. Plane P contains the points A(0, -5, 6) and B(4, 3, -2), and the vector $\vec{u} = \langle -3,1,4\rangle$.

a) Find vector $\overrightarrow{AB}$. [2 pt]

$\overrightarrow{AB} = \langle 4, 8, -8\rangle$.

b) Use vector $\overrightarrow{AB}$ and vector $\vec{u}$ to write a vector equation for plane P. [3 pts]

$\langle x, y, z\rangle = \langle 0, -5, 6\rangle + t\langle 4, 8, -8\rangle + s\langle -3, 1, 4\rangle$.

$10x + 2y + 7z = 32$

c) Plane Q is a plane that is a perpendicular bisector of line segment AB. Write the equation for plane Q in standard rectangular form: $ax + by + cz = d$. [4 pts]

plane Q bisects at $(2, -1, 2)$

$4x + 8y - 8z = -16$

$\boxed{x + 2y - 2z = -4}$

3. Circle "TRUE" or "FALSE" for each statement. Given that $(\vec{a}\times\vec{b})\bullet\vec{c} = 0$ ... [1 pt each]

a) $\vec{a}$ and $\vec{b}$ must be orthogonal TRUE or FALSE

b) $\vec{a}$ and $\vec{c}$ must be orthogonal TRUE or FALSE

c) $(\vec{a}\times\vec{b})$ and $\vec{c}$ must be orthogonal TRUE or FALSE

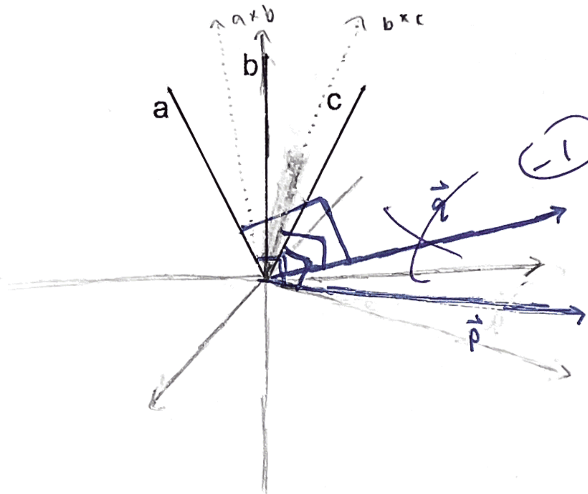
d) $\vec{a}$, $\vec{b}$ and $\vec{c}$ must be coplanar TRUE or ~~FALSE~~

-1

4. **STATEMENT 1: For vectors, the Associative Property is FALSE** (that is, $(\vec{a}\times\vec{b})\times\vec{c}\neq\vec{a}\times(\vec{b}\times\vec{c})$).

In the diagram below, vectors **a**, **b**, and **c** are all have the same magnitude, and the angle between **a** and **b** is equal to the angle between **b** and **c**.

Let $\vec{p}=(\vec{a}\times\vec{b})\times\vec{c}$ and $\vec{q}=\vec{a}\times(\vec{b}\times\vec{c})$. Draw and label vectors p and q on the diagram, to prove **STATEMENT 1**. (You'll be graded on the directions of your answers, and their relative magnitudes). [4 pts]



$\hat{i}$ $\hat{j}$ k
5 0 0
0 5 0

25

5. Consider the plane 2x – 5y + 8z -13 = 0, with a line that is perpendicular to the plane. The line also passes through the point (-4, 1, 7).

a) Write the equation of the line in parametric form. [3 pts]

$\langle x, y, z\rangle = \langle -4, 1, 7\rangle + t\langle 2, -5, 8\rangle$

$x = -4 + 2t$
$y = 1 - 5t$
$z = 7 + 8t$

b) How far is the point (10, 4, 2) from the plane? [3 pts]

$$\frac{|20-20+16-13|}{\sqrt{4+25+64}} = \boxed{\frac{3}{\sqrt{93}}}$$

6. A plane is defined by the equation $\langle x,y,z\rangle=\langle -3,2,4\rangle+s\langle -1,0,1\rangle+t\langle 0,5,1\rangle$. Write the equation of the plane in standard form: $ax+by+cz=d$. [3 pts]

~~$x = -3 - s$~~
~~$y = 2 + 5t$~~
~~$z = 4 + s + t$~~

x y z
~~$a(-3-s)+b(2+5t)+c(4+s+t)=0$~~

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & 1 \\ 0 & 5 & 1 \end{vmatrix} = -5i + j - 5k$$

$\boxed{-5x + y - 5z = -3}$

~~$15 \quad 2 + 20$~~

$15 + 2 - 20$

-1