Analysis H 2021-2022 – Hahn / Tantod
Unit 8: Sequences and Series Quiz 2
No Calculators!

47
50 points

Why so series? Chris Lee
Period: 3

#1-5 are Multiple Choice: Circle the best answer. [2 pts each]

1. Which of these is NOT a test for convergence for series?

   a) ratio test b) nth term test c) geometric series test c) comparison test

   e) none of the above (all the listed answers can be used to test for convergence)

2. Consider the explicit definition: $a_n = (n-5)^2 + 8$. For this definition, the **sequence** is ...

   a) ~~everywhere increasing~~ b) ~~everywhere decreasing~~ c) increasing only for n > 5

   d) decreasing only for n > 5 e) ~~strictly alternating~~

3. Consider the explicit definition: $a_n = (n-5)^2 + 8$. For this definition, the **sequence of partial sums** is ...

   a) everywhere increasing b) everywhere decreasing c) increasing only for n > 5

   d) decreasing only for n > 5 e) strictly alternating

   24, 51

4. Which of the following is a subsequence of $\{n(-1)^{n+1}\}$? (Which of the following sets are entirely contained within the given sequence?)

   a) natural numbers b) negative integers c) rational numbers

   d) positive odd numbers e) positive even numbers

5. $\sum \frac{1}{n^p}$ will diverge for what values of p?

   a) $p > 0$ b) $p \geq 1$ c) $p > 1$ d) $p < 1$ e) $p \leq 1$

6. A certain sequence $\{a_n\}$ converges to L with a neighborhood of ε. Write "True" or "False" for each. [1 pt each]

   a) For all values of n, $L - \varepsilon \leq a_n \leq L + \varepsilon$ false

   b) There are an infinite number of values for a_n that are inside the neighborhood. true

   c) If a_n is within the neighborhood, then a_{n+1} is also in the neighborhood. ~~true~~

   d) There is a value of M such that if $n \geq M$, a_n is within the neighborhood. true

   e) If the sequence is everywhere decreasing, then it is bounded below by L. true

-1

7. For each **sequence**, write "C" for converge or "D" for diverge. You don't have to prove them. [1 pts each]

a) $a_n = \frac{1}{n}$

C

b) $a_n = \frac{1}{n^2}$

C

c) $a_n = \frac{1}{\sqrt{n}}$

$\frac{1}{\sqrt{1}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{4}} \ldots$

C

d) $a_n = \frac{n^n}{n!}$

D

e) $a_n = \sin n$

D

f) $a_n = \frac{1}{(1.2)^n}$

C

g) $a_n = \frac{1}{(0.8)^n}$

D

h) $a_n = \frac{n+1}{(-1)^n}$

D

8. For each infinite **series**, write "C" for converge or "D" for diverge. You don't have to prove them. [1 pt each]

a) $a_n = \frac{1}{n}$

D

b) $a_n = \frac{1}{n^2}$

C

c) $a_n = \frac{1}{\sqrt{n}}$

D

d) $a_n = \frac{n^n}{n!}$

D

e) $a_n = \sin n$

D

f) $a_n = \frac{1}{(1.2)^n}$

C

g) $a_n = \frac{1}{(0.8)^n}$

D

h) $a_n = \frac{n+1}{(-1)^n}$

D

9. Write a series that converges to 15. Give your answer in Sigma notation. [4 pts]

$\boxed{\cancel{\sum_{n=1}^{\infty} 15n}}$

-2

-2

alt series, ratio, geometric, nth term, comparison

Questions 10-12: [5 pts each]
For each series, write a clear proof to show convergence or divergence, first indicating the name of the test you used. **Important: for these 3 problems, you MAY NOT use the same test twice!** If you use the same test for more than one problem, you will lose 3 points per infraction. Do the work for #10 and 11 on this page, and then the work for #12 on the back of this page.

10. $\sum_{n=2}^{\infty} \frac{cos(n\pi)}{\sqrt{n}}$ Test used: alternating series

✓ strictly alt: $\cos(n\pi)$ can only be 1 or -1.

✓ abs value decreasing: $\frac{1}{\sqrt{n}} \geq \frac{1}{\sqrt{n+1}}$ $\sqrt{n+1} \geq \sqrt{n}$

$n+1 \geq n$

$1 > 0$ ALWAYS TRUE

✓ nth term goes to 0

↓

$\lim_{n\to\infty} \left\{\frac{\cos(n\pi)}{\sqrt{n}}\right\} = 0$ since top stays -1 or 1, denom gets bigger and bigger

$\boxed{\therefore \left\{\frac{\cos(n\pi)}{\sqrt{n}}\right\} \text{ converges}}$

11. $\sum_{n=1}^{\infty} \frac{2^n}{3^n + n}$ Test used: ratio test

$\lim_{x\to\infty} \left| \dfrac{\frac{2(2^n)}{3(3^n)+(n+1)}}{\frac{2^n}{3^n+n}} \right| = \lim_{x\to\infty} \left| \frac{2(\cancel{2^n})}{3(3^n)+(n+1)} \cdot \frac{3^n+n}{\cancel{2^n}} \right| = \lim_{x\to\infty} \left| \frac{2\cdot 3^n + n}{3\cdot 3^n + n+1} \right|$

↓

3^n increases faster so the "n" on top and bottom become insignific since there's a "3^n" on both top and bottom, take ratio

$\lim_{x\to\infty} = \frac{2}{3}$

$\frac{2}{3} < 1$

$\boxed{\therefore \left\{\frac{2^n}{3^n+n}\right\} \text{ converges}}$

12. $\sum_{n=1}^{\infty} \frac{6 + 8n + 9n^2}{3 + 2n + n^2}$ Test used: nth term test
(Show your work for this problem on the back of this page)

800 90000 10,000

3 + 200

$\frac{23}{6}$

$\frac{90806}{10203}$

12. $\sum_{n=1}^{\infty} \frac{6+8n+9n^2}{3+2n+n^2}$ Test used (write it again): nth term test

Let $n = 100$ for $\left\{ \frac{6+8n+9n^2}{3+2n+n^2} \right\} = \frac{90806}{10203}$

$\downarrow$

in order for series to converge, nth term must approach 0 but here it approaches 9.

$\boxed{\therefore \left\{ \frac{6+8n+9n^2}{3+2n+n^2} \right\} \text{ diverges}}$