Analysis H 2021-2022 – Hahn / Tantod
GAtM Quiz 2
No Calculators!

35.5 / 36 points

Looking for Group: Michelle Goo? Goo

Period: 6

1. For each group named below, write the order of the group. If the group is infinite, write either "C" for "countable infinity" or "U" for "uncountable infinity". [1 pt each]

A) cyclic group of a hexagon 6

B) dihedral group of a triangle 6

C) rotation group of a square prism 8

D) addition of rational numbers C

E) addition of integers C

F) rotation group of a pyramid with a hexagon base 6

G) dihedral group of a square 8

H) rotation/reflection group of a cube 48

I) addition of positive and negative even numbers C

J) addition of complex numbers U

2. Which of the groups from #1 are isomorphic to each other? Separate the letters of different isomorphic sets with brackets, like this: {A and J}, {C, D, and H}, ... [2 pts]

{A & F} {C & G}. ~~{D & E}~~

~~{B & E & I}~~ - {I}

3. Write "True" or "False" for each statement. [1 pt each]

a) Drawing a one-to-one correspondence between two sets proves they have the same cardinality T

b) Sets with the same cardinality are isomorphic F

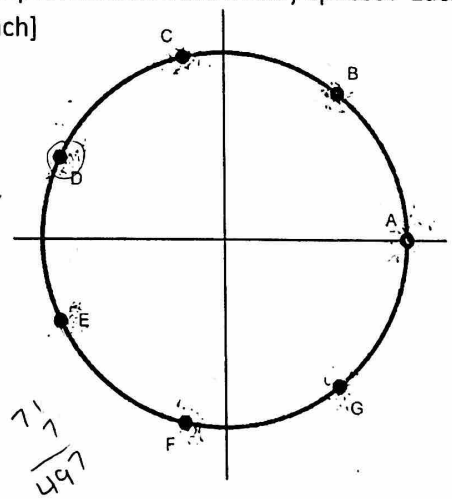
c) Countable infinities are the same size as uncountable infinities F

d) Given a list of real numbers [0, 1), Cantor's diagonal argument can create a new unique real number T

e) There are more points on a line than there are on a line segment F

f) There are more positive fractions than there are positive multiples of 9. F

4. The circle on the complex axis below has radius 1, and the 7 complex numbers are evenly spaced. Each of your answers for these questions should be a letter from A – G. [1 pt each]



a) AG = ~~B~~ G

b) BC = D

c) DF = ~~A~~ B

d) E^3 = F

e) C^{500} = ~~D~~ G

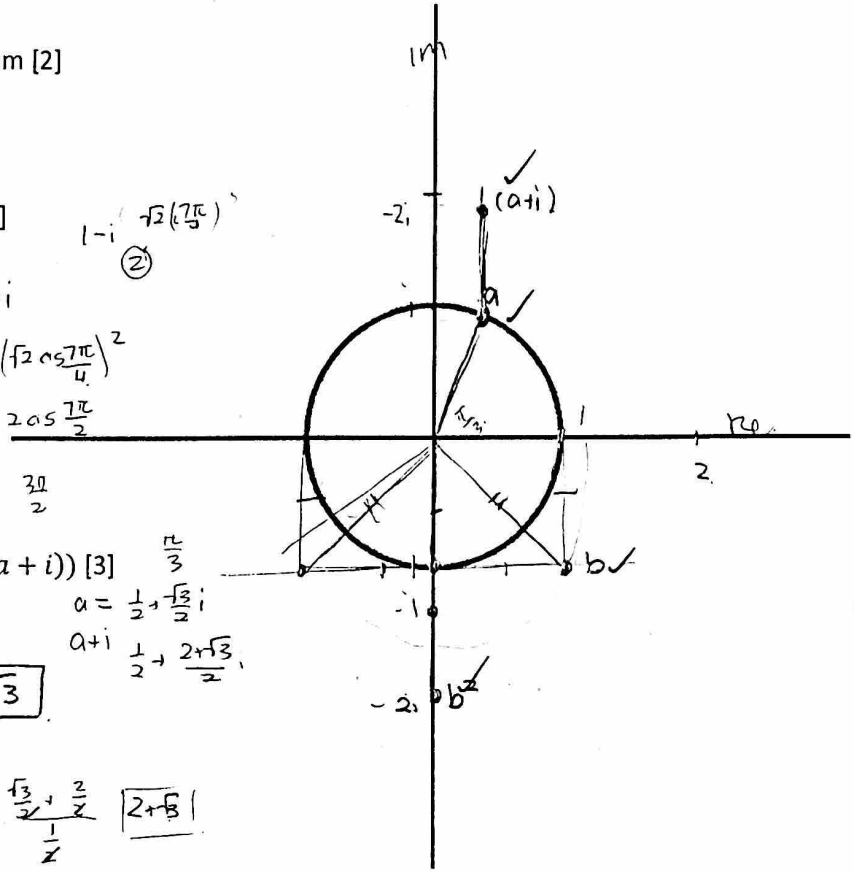
5. The circle on the complex axis below has a radius 1.

a) Given $|a| = 1$ and $Arg(a) = \frac{\pi}{3}$,

draw and label a and $(a + i)$ onto the diagram [2]

b) Given $|b| = |b - 2|$ and $Im(b) = -1$,

draw and label b and b^2 onto the diagram [2]



$|b| = |b-2|$ $\quad a+bi$

$1-i \quad -1-i$

$1-i \quad -2i \quad -1-i$

$-1-i \quad \left(\sqrt{2}\,cis\frac{7\pi}{4}\right)^2$

$2\,cis\frac{7\pi}{2} \quad 2\,cis\frac{3\pi}{2}$

$\frac{4\pi}{2}, \frac{3\pi}{2} \quad 2\,cis\frac{7\pi}{2} \quad 4\pi \quad \frac{3\pi}{2}$

$1-i \quad \sqrt{2}\left(\frac{7\pi}{4}\right)$ (2)

$\left(\sqrt{2}\,cis\frac{7\pi}{4}\right)^2$

c) find a simplified expression for $\tan(Arg(a + i))$ [3]

$\frac{\pi}{3}$

$a = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

$a+i \quad \frac{1}{2} + \frac{2+\sqrt{3}}{2}i$

$\tan(arg(a+i)) \quad \frac{\frac{2+\sqrt{3}}{2}}{\frac{1}{2}} = \boxed{2+\sqrt{3}}$

$= \frac{\sin}{\cos} = \frac{\frac{2+\sqrt{3}}{2}}{\frac{1}{2}}$

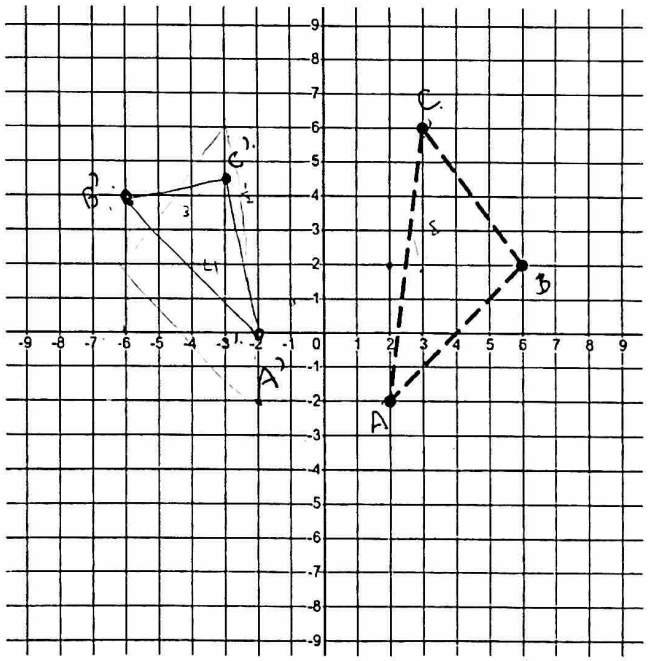
$\frac{\sqrt{3}}{2} + \frac{2}{2} \quad \frac{1}{2} \quad \boxed{2+\sqrt{3}}$

$\tan\left(\frac{1}{2} + \left(\frac{2+\sqrt{3}}{2}\right)i\right) \quad 2+\sqrt{3}$

$\begin{bmatrix} -1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

6. A set of pre-image points are graphed on the coordinate axis below.

a) Transform the points according to the matrix $T = \begin{bmatrix} -1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$, and graph the image. Show the matrix multiplication you used to find your answer. [3]



$\begin{bmatrix} -1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 2 & 6 & 3 \\ -2 & 2 & 6 \end{bmatrix} = \begin{bmatrix} -2 & -6 & -3 \\ 0 & 4 & \frac{9}{2} \end{bmatrix}$

$1-1 \quad 3+1 \quad \frac{3}{2}+3 \quad \frac{3}{2}+\frac{6}{2}$

$1-1 \quad 3+1 \quad \frac{3}{2}+3 \quad \frac{9}{2} \quad 4\frac{1}{2} \quad \frac{9}{2}$

$\begin{bmatrix} -1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{bmatrix} 2 & 6 & 3 \\ -2 & 2 & 6 \end{bmatrix} = \begin{bmatrix} -2 & -6 & -3 \\ 0 & 4 & \frac{9}{2} \end{bmatrix}$

b) Using specific math vocabulary, describe the effect of the transformation T in a few words. [2]

a flip over the y-axis. ~~& a dilation~~

-0