Analysis H – Hahn / Tantod
Unit 1: AtPS, Quiz 2
NO CALCULATORS

19/20

I choose you: Chris Lee
Period: 3

1. Use mathematical induction to prove that the given formula works for all positive integers n. [6 points]

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

1) prove $n=1$ is true

$1^3 = \frac{1^2(1+1)^2}{4}$.

2) assume $n=k$ is true

$1^3 + 2^3 + \ldots + k^3 = \frac{k^2(k+1)^2}{4}$.

3) prove $n=k+1$ is true

$1^3 + 2^3 + \ldots k^3 + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4} = \frac{k^2(k+1)^2 + 4(k+1)^3}{4} = \frac{(k+1)^2(k+2)^2}{4}$

$\frac{k^2(k^2+2k+1) + 4(k^3+2k^2+k+k^2+2k+1)}{4} = \frac{(k^2+2k+1)(k^2+4k+4)}{4}$

$(k^4+2k^3+k^2+4k^3+8k^2+4k+4k^2+8k+4)/4 = (k^4+4k^3+4k^2+2k^3+8k^2+8k+k^2+4k+4)/4$

$(k^4+6k^3+13k^2+12k+4)/4 = (k^4+6k^3+13k^2+12k+4)/4$ ✓

2. Use mathematical induction to prove that $n^3 - n$ is divisible by 6 for all $n \geq 2$. [6 points]

1) base case: prove $n=2$ is true

$2^3 - 2 = 6$ ← div by 6.

2) assume $n=k$ is true

$k^3 - k$ is divisible by 6.

$(k+1)(k+1)$

$k+1\,(k^2+2k+1)$

3) prove $n=k+1$ is true

$(k+1)^3 - (k+1)$ is div by 6

$k^3 + 2k^2 + k + k^2 + 2k + 1 - k - 1 = (k^3 - k) + 3k^2 + 3k$ ✓

div by 6 from step 2

both terms are divisible by 3 since they're mult by 3

~~adding 2 terms that are div by 3 means they can be div by 6~~

-1

not true: 6+9=15

adding two terms both div by 6 means final sum also div by 6

-1

3. Simplify: [4 points]

$$\frac{(2n+2)!\,(n!)^2}{[(n+1)!]^2(2n)!}$$

$$\frac{(2n+2)!\,(n!)^2}{[(n+1)!]^2(2n)!}$$

$$\frac{(2n+2)\cdot(2n+1)\,(n!)^2}{[(n+1)!]^2}$$

$$\frac{(2n+2)(2n+1)}{(n+1)\,(n+1)}$$

$$\cancel{\frac{(2n+2)(2n+1)}{(n+1)(n+1)}}$$

$$\boxed{\frac{2(2n+1)}{(n+1)}}$$

4. Evaluate: [2 points each]

$\binom{n}{k}$ $\frac{n!}{k!(n-k)!}$

a) $\binom{-2}{10} = 11$

$$\frac{-2\cdot -13\cdot -4\ldots}{10!\,(-12)!}$$

$$\frac{+\cancel{2}\cdot +\cancel{3}\cdot +\cancel{4}\cdot +\cancel{5}\cdot +\cancel{6}\cdot +\cancel{7}\cdot +\cancel{8}\cdot +\cancel{9}\cdot +\cancel{10}\cdot +11}{\cancel{10}\cdot\cancel{9}\cdot\cancel{8}\cdot\cancel{7}\cdot 6\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot 8\cdot 1}$$

b) $\binom{-4}{3} = -20$

$$\frac{-4\cdot -5\cdot -6\cdot -7\ldots}{3!\,(-7)!}$$

$$\frac{+\overset{2}{\cancel{4}}\cdot -5\cdot +\overset{2}{\cancel{6}}}{\cancel{3}\cdot\cancel{2}\cdot 1}$$

/-0