Analysis H –Hahn/Tantod
Calculus Ch 3 Quiz 2022
NO CALCULATORS (33 pts)

32/33

Michelle Koo is the formal definition of awesome
Period: 6

1. Use the Intermediate Value Theorem to prove that $f(x) = x^5 - x - 2$ has a solution in the x-interval [0, 2]. (4 pts)

$f(x)=x^5-x-2$ has solution in x-interval $[0,2]$

① $f(x)$ is a continuous function

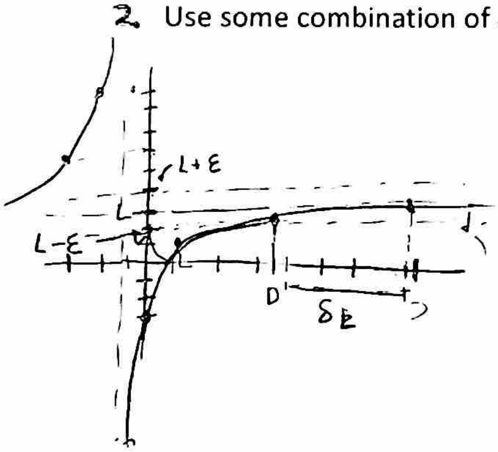
② $f(0)=-2$

$f(2)=32-2-2=28$

because every y-value between -2 & 28 is possible between x-interval [0,2], and function is continuous, it must pass through a point when $y=0$. Therefore $f(x)$ has a solution in the x-interval [0,2].

2. Use some combination of $\delta, \varepsilon, D,$ *and* E to prove: $\lim_{x\to\infty} \frac{6x-3}{2x+1} = 3$. Include a graph in your proof. (5 pts)

$f(x)=$



$\lim_{x\to\infty} \frac{6x-3}{2x+1} = L =$

$\lim_{x\to\infty} f(x)=L$ iff $\forall E>0, \exists D>0$

$x \geq D$ ~~iff~~ $|f(x)-E|<E$

below

so $\underline{L-E}$ = yvaluve for $f(D)$

$\frac{6x-3}{2x+1} = 3-\varepsilon$ for $x=D$

$\frac{6D-3}{2D+1} = 3-\varepsilon$

$6D-3=(2D+1)(3-\varepsilon)$

$6D-3=6D-2D\varepsilon+3-\varepsilon$

$2D\varepsilon=6-\varepsilon$

$D=\frac{6-\varepsilon}{2\varepsilon}$

x	y
0	$\frac{-3}{1}$
-1	$\frac{-9}{-1}=9$
-2	$\frac{-12-3}{-4+1}=\frac{-15}{-3}=5$
1	$\frac{6-3}{2+1}=1$

For a small # $\varepsilon>0$, there exists $D>0$, $D=\frac{6-\varepsilon}{2\varepsilon}$ such that

$x \geq \frac{6-\varepsilon}{2\varepsilon} \rightarrow |f(x)-3|<\varepsilon$

$\therefore \lim_{x\to\infty} \frac{6x-3}{2x+1} = 3$

$3-\varepsilon = \frac{6x-3}{2x+1}$

$(3-\varepsilon)(2x+1)=6x-3$

$6x+3-2\varepsilon x-\varepsilon=6x-3$

$\frac{6-\varepsilon}{2\varepsilon}=x$

1 - D

3. Given the function $y = x^2 - 8x - 7$,

$2x-8$

$2 \quad 4-8 = -4$

a) Use the **definition of derivative at a point** to calculate the derivative at x = 2. Make sure you use proper limit notation throughout. (3 pts)

$f'(x) = \lim_{x \to c} \frac{f(x)-f(c)}{x-c}$ $\quad c=2$

$= \lim_{x \to 2} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x \to 2} \frac{x^2-8x-7+19}{x-2}$ $\quad 6 \; 2$

$= \lim_{x \to 2} \frac{x^2-8x+12}{x-2}$ $\quad \lim_{x\to 2} x-6 \quad 2-6$

$= \lim_{x \to 2} \frac{(x-6)(x-2)}{(x-2)} = \boxed{-4}$

$f(2) = (2)^2 - 8(2) - 7$

$= 4 - 16 - 7$

$= -12 - 7 = -19$

$\boxed{\text{derivative is } -4}$

25

$f(x) \quad x^2-8x-7-4+18+7$

b) Find $\frac{dy}{dx}$ (any way) (2pts)

$y = x^2-8x-7$ $\quad$ powerrule

$\frac{dy}{dx}\, x^2-8x-7 = \boxed{2x-8}$

$at \; x-2$

-4

c) Find the equation of the line tangent to y and parallel to $2x + y = 4$. (3 pts)

$f'(x) = 2x-8 = -2$ $\quad$ ← $\quad y = -2x+4$ $\quad$ slope $= -2$

$2x = +2+8 = 6$

$x = 3$

$y = x^2-8x-7$ $\quad (3,-22)$

$= 9-8(3)-7$ $\quad -31$

$= 9-24-7$

$= -15-7 = -22$

$\boxed{y+22 = -2(x-3)}$

$2x-8 = -2$

$2x = 6$

$x = 3$

$9-24-7$

$9-31$

-22

4. Use the **formal definition of the derivative** to find the derivative of the function $y = \frac{1}{5x+3}$. Use proper limit notation throughout. (3 pts)

$(5x+5h+3)(5x+3)$

$25x^2+15x+25xh+15h+15x+9$

$f(x) = y = \frac{1}{5x+3}$

$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{\frac{1}{5x+5h+3} - \frac{1}{5x+3}}{h}$

$= \lim_{h\to 0} \frac{\frac{5x+3-5x-5h-3}{(5x+5h+3)(5x+3)}}{h}$

$= \lim_{h\to 0} \frac{-5h}{h(25x^2+30x+9+25hx+15h)}$

$= \frac{-5}{25x^2+30x+9}$

$= \lim_{h\to 0} \frac{-5}{25x^2+30x+9+25xh+15h}$

$(5x^{-}+3)^{-1}$

$\frac{dz}{dy} = \boxed{\frac{-5}{25x^2+30x+9}}$

$\frac{1}{5x+3}$

$\frac{1}{-5}$

$(5x+5h+3)(5x+3)$

$25x^2+15x+25hx+15h+15x+9$

$25x^2+30x+9+25hx+15h$

$(5x+5h+3)(5x+3)$

$25x^2+25xh+15x+15x+15h+9$

$25x^2+30x+9+25xh+15h$

-D

5. Find the value(s) of a and b that will make the function continuous. (4pts)

$$f(x) = \begin{cases} \dfrac{(x^2-4)}{x-1} & if\ x < 2 \\ ax^2 - bx + 3 & if\ \ 2 \le x < 3 \\ 2x - a + b. & if\ x \ge 3 \end{cases}$$

$\frac{(x^2-4)}{x-1} = ax^2 - bx + 3$ when $x = 2$.

$\frac{(4-4)}{2-1} = 4a - 2b + 3.$

$0 = 4a - 2b + 3.$ ✓

$2b - 3 = 4a$ $a = \frac{2b-3}{4}$

$a = \frac{2(\frac{9}{2})-3}{4}$

$= \frac{6}{4} = \frac{3}{2}$

$ax^2 - bx + 3 = 2x - a + b$ when $x = 3.$

$9a - 3b + 3 = 6 - a + b.$ ✓

$10a = 4b - 3$

$\cancel{10}^{5}\left(\frac{2b-3}{\cancel{4}_2}\right) = \frac{10b-15}{2} = 4b - 3$

$10b - 15 = 8b - 6$

$2b = 9$

$b = \frac{9}{2}$

$\boxed{a = \frac{3}{2}\ \cancel{b = \frac{9}{2}}}$

-1

$4a - 2b + 3 = 0$

$9a - 3b + 3 = 6 - a + b$

$10a = \cancel{3}\ 4b - 3$

$5\ \cancel{10}\left(\frac{2b-3}{\cancel{4}}\right)$ $10b - 15 = 6 + 8b$

$2b = 9$

0

$\frac{3}{2}(\cancel{4}^2) - \frac{9}{2}(2) + 3$

$6 - 9 + 3 = 0$

$\frac{3}{2} \times 1 - \cancel{9+3}$ $6+3$

$\frac{3}{2}(9) - \frac{9}{2}(3) + 3$

$\frac{27-27}{2} + 3 = 3$

$\frac{27}{2} - \frac{27}{2}\ 3$

$2(3) - \left(\frac{3}{2} + \frac{9}{2}\right)$

$6 - \frac{6}{2} = 3$

$6 - \frac{3}{2}, \frac{9}{2}$ $\frac{6}{2}$

6. Given that $\lim_{x\to 0}\frac{\sin x}{x} = 1$ and $\lim_{x\to 0}\frac{\cos x - 1}{x} = 0$, use the formal definition of the derivative to find $\frac{d}{dx}(\cos x)$. (4 pts)

FDoDa

$\frac{d}{dx}(\cos x) = \lim_{h\to 0}\frac{\cos(x+h) - \cos x}{h}$

$= \lim_{h\to 0}\frac{\cos x\cos h - \sin x\sin h - \cos x}{h}$

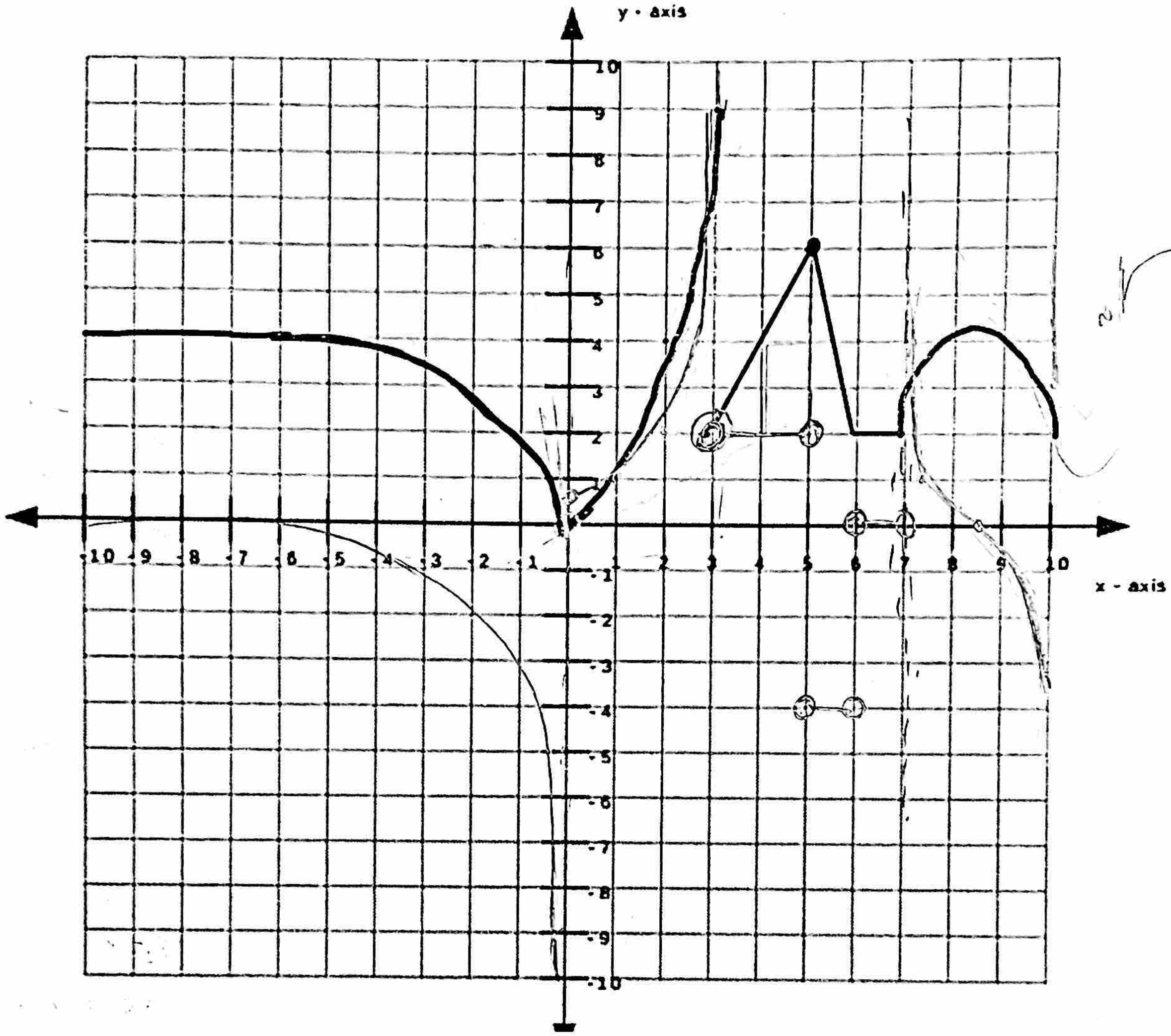
$= \lim_{h\to 0}\left(\cos x\frac{(\cos h - 1)}{h} - \sin x\left(\frac{\sin h}{h}\right)\right)$

$= \cancel{\cos x\left(\lim_{h\to 0}\left(\frac{\cos h - 1}{h}\right)\right)}^{0} - \sin x\left(\cancel{\lim_{h\to 0}\left(\frac{\sin h}{h}\right)}^{1}\right)$

$= \boxed{-\sin x}$.

-1

7. f(x) is given below. On the same graph, **accurately** sketch f '(x). Note: there is a cusp at x=0. (5 pts)



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