Analysis H – Hahn / Hlasek / Tantod
MT Exam 2: Polar, 3D, parametric, vectors
No Calculators

95
100 points

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Period: 7

49/50 46/50

Part 1: Polar Graphing

Questions 1-4 are Multiple Choice. Circle the best answer. [2 pts each]

1. In the polar system, the graph of $r = 7\cos 8\theta$ is a _____.

   a) Rose curve with 8 petals (b) Rose curve with 16 petals c) Rose curve with 7 petals

   d) Lemniscate symmetric about the line $\theta = 0$ e) None of the above

2. In the polar system, the graph of $r^2 = 32\sin 2\theta$ is a _____.

   a) Rose curve with 2 petals b) Rose curve with 4 petals c) Rose curve with 32 petals

   d) Lemniscate symmetric about the line $\theta = 0$ (e) None of the above

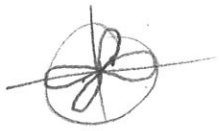
3. In the polar system, the graph of $r = 4 - 8\sin\theta$ is a _____.

   (a) Limacon with an inner loop b) Dimpled Limacon c) Cardioid

   d) Convex Limacon e) None of the above

4. In the Polar System, the graphs of $r^2 = 4\sin 2\theta$ and $r^2 = 4\cos 2\theta$ have _____ points of intersection.

   a) less than 3 (b) 3 c) 4 d) 5 e) More than 5



$\frac{\pi}{4}$, $r^2 = 4\sin\frac{\pi}{4} = 2\sqrt{2}$
r^2

Free Response: Show all your work to receive full credit.

5. Convert the polar equation to rectangular form. Write your answer as a function y in terms of x. [3 pts]

$r = \frac{2}{1-\sin\theta}$

$r(1-\sin\theta) = 2$

$r - r\sin\theta = 2$

$\sqrt{x^2+y^2} - y = 2$

$x^2+y^2 = (y+2)^2$

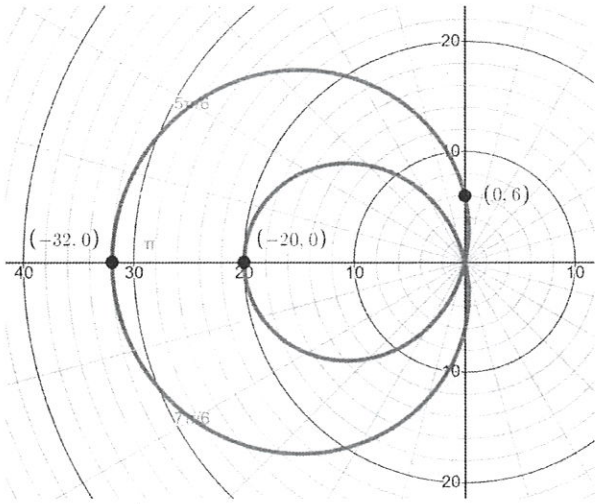
$x^2+y^2 = (y^2+4y+4)$

$y = \frac{x^2}{4} - 1$

-0

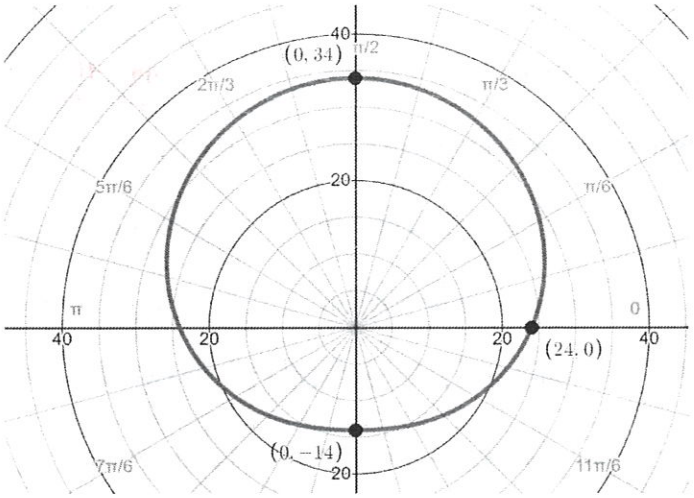
6. On the graphs below, the labeled points are all in rectangular form. Write a **polar** equation to match each graph. Write each answer as a function r in terms of theta. [3 pts each]

a)



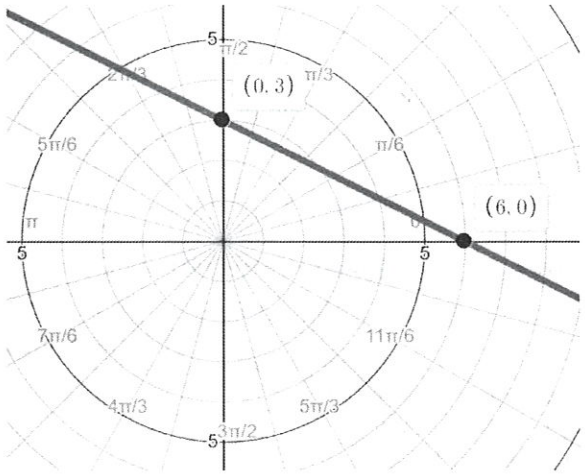
Equation: $r = 6 - 26\cos\theta$

b)



Equation: $r = 24 + 10\sin\theta$

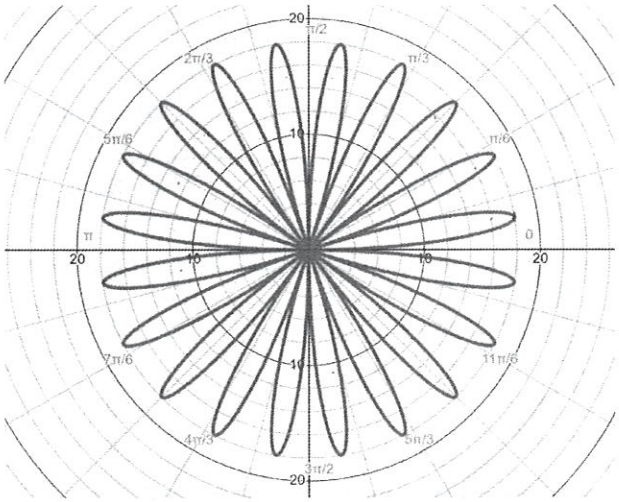
c)



Equation: $r = \frac{6}{2\sin\theta + \cos\theta}$

$y = -\frac{x}{2} + 3 \qquad r\sin\theta = -\frac{r\cos\theta}{2} + 3 \qquad r\left(\sin\theta + \frac{\cos\theta}{2}\right) = 3$

d)



Equation: $r = 18\sin 10\theta$

7. Give the polar coordinates of the tip of the 1st petal in the graph of 6(d) above. (By the "1st petal", we mean the petal that is in the first quadrant, closest to the positive x-axis). [2 pts]

$\sin 10\theta = 1$

$\theta = \frac{\pi}{20}$

$\left(18, \frac{\pi}{20}\right)$

Part 2: 3D Graphing

Questions 1-4 are Multiple Choice. Circle the best answer. [2 pts each]

1. In the 3D system, the graph of $y^2 - \frac{x^2}{6} - z = 0$ is a _____.

   a) Parabolic Cylinder b) Elliptic Cylinder c) Hyperbolic Cylinder

   d) Hyperbolic Paraboloid e) Elliptic Paraboloid

2. In the 3D system, the graph of $\frac{x^2}{3} + \frac{y^2}{4} = \frac{z^2}{5}$ is a _____.

   a) Elliptic Cone b) Elliptic Paraboloid c) Hyperboloid of 1 sheet

   d) Hyperboloid of 2 sheets e) Hyperbolic Paraboloid

3. In the 3D system, the **intersection** of the graphs of $y = x^2 + z^2$ and $x = 10$ is a _____.

   a) Ellipse, but not a Circle b) Circle c) Parabola

   d) Hyperbola e) Plane

4. In the 3D system, the **intersection** of the graphs of $x^2 + z^2 = 25$ and $2x + 3y + 4z = 12$ is _____.

   a) Ellipse, but not a Circle b) Circle c) Parabola

   d) Hyperbola e) Plane

Free Response: Show all your work to receive full credit.

5. Consider the graph of $\frac{x^2}{9} + y^2 - 8y + z^2 = 25$ in the 3D system.

   a) Classify the quadric surface by its most specific name [2 pts]

   $\frac{x^2}{9} + y^2 - 8y + z^2 = 25$

   $\frac{x^2}{9} + (y-4)^2 + z^2 = 41$

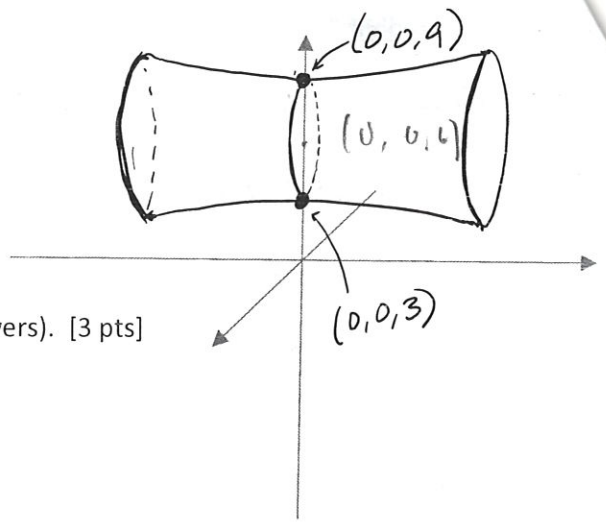
   ellipsoid

   b) Find the y-intercept(s) of the graph. [3 pts]

   $(y-4)^2 = 41$

   $y = 4 \pm \sqrt{41}$

6. Consider the 3D sketch of a quadric surface on the right.



a) Classify the quadric surface by its most specific name. [2 pts]

hyperboloid of 1 sheet

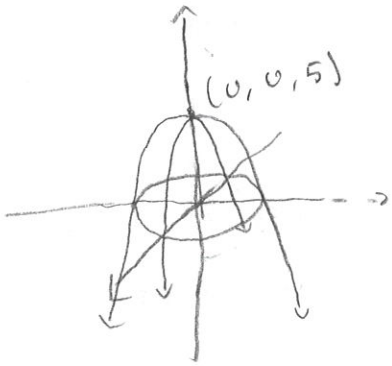
b) Write an equation that matches the graph (many possible answers). [3 pts] -1

$(z-3)^2+x^2=9$

$(z-\frac{3}{6})^2+x^2-y^2=9$

7. A certain quadric surface has an xy-trace of an ellipse, an yz-trace of a parabola, and a xz-trace of a parabola (the xz-trace parabola is skinnier than the yz-trace parabola). The quartic surface has a z-intercept at (0, 0, 5).

a) Sketch the quadric surface. [2 pts]



b) Classify the quadric surface by its most specific name. [2 pts]

elliptic paraboloid

c) Write an equation that matches your graph (many possible answers). [3 pts]

$2x^2+y^2=-(z-5)$

$2x^2+y^2=-z+5$

$2x^2+y^2+z=5$

-1

$\sin^2 = 1 - \cos^2$

$x = 3 - \sin^2\left(\cos^{-1}\left(\frac{1}{2}\right)\right)$

Part 3: Vectors and Parametric Equations

Questions 1-4 are Multiple Choice. Circle the best answer. [3 pts each]

$y = 2\cos t$

$t = \cos^{-1}\left(\frac{1}{2}\right)$

$\cos t = \frac{1}{2}$

$x = 3 - \left[1 - (\frac{1}{2})^2\right]$

1. The graph of the set of parametric equations $\begin{matrix} x(t) = 3 - sin^2 t \\ y(t) = 2cost \end{matrix}$ is a _____.

a) circle b) ellipse c) parabola d) hyperbola e) spiral

2. Given: $i, j,$ and k are standard unit vectors. Find $|j - k|$.

a) j b) -j c) <1, 0, -1> d) $\sqrt{2}$ e) $-\sqrt{2}$

3. If $\vec{u}$ goes from the door to the TV, and $\vec{v}$ goes from the TV to the teacher's desk, which of the following directions BEST describes $\vec{u} \times \vec{v}$?

a) from the ceiling to the floor b) from the floor to the ceiling

c) from the TV to the door d) from the TV to the teacher's desk

e) from the door to the teacher's desk

4. The equations of two lines are given below in parametric form. The two lines are _____.

$$L_1 = \begin{cases} 2t - 4 \\ 5t + 3 \\ -7t - 2 \end{cases} \quad L_2 = \begin{cases} -4t + 2 \\ -10t + 2 \\ 14t + 3 \end{cases}$$

$2t - 4 = -4t + 2$

$6t = 6$

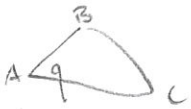
$t = 1$

a) skew b) parallel c) intersecting but not perpendicular

d) intersecting and perpendicular e) the same line

-3

Questions 5-8 are Multiple Choice. Circle the best answer.



5. If the area of triangle ABC = 10, and Angle A = q, then $\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\sin q =$

a) 20 b) 5 c) 40 d) 10 e) not enough info

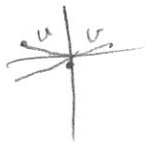
6. Find $(k \times -j) \times j$

a) *i* b) *-i* c) *k* d) *0* e) *-k*

$\hat{i}\ \hat{j}\ \hat{k}$

7. In 2-D, consider $\vec{u} =< -7,2 >$ and $\vec{v} =< 5,1 >$. Which of the following most closely resembles $proj_v\vec{u}$?

a) b) c) d) e)



8. If a ball is launched at a 15-degree angle of elevation at a velocity of 80 feet per second, which function most closely represents the height of the ball after t seconds?

a) $h = -16t^2 + 80t$ b) $h = -\frac{1}{2}t^2 + 20t$ c) $h = -16t^2 + 60t$

d) $h = --\frac{1}{2}t^2 + 60t$ e) $h = -16t^2 + 20t$

$\sin\theta \approx \theta$, $\sin\theta \approx \frac{\pi c}{12} \approx \frac{1}{4}$

Free Response

9. Find the **rectangular** equation of the plane through (3, 2, 4) that will be perpendicular to the pair of intersecting planes $3x + 2y - z = 6$ and $x - 2y - z = 4$. [4 pts]

$3x+2y-z=6$
$+)\ x-2y-z=4$
$4x-2z=10$

$(0, \frac{1}{2}, -5)$
$(1, 0, -3)$

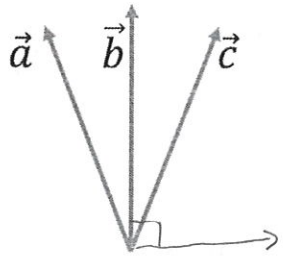
$3x+2y-z=6$
$-)\ x-2y-z=4$
$2x+4y=2$

$\langle 1, -\frac{1}{2}, 2 \rangle$
$\langle 2, -1, 4 \rangle$

$2x-y+4z=D$
$2(3)-(2)+4(4)=D$
$D=20$

$2x-y+4z=20$

10. In the figure below, all three vectors are unit vectors. Draw the resultant vector of $\vec{b} \times (\vec{c} \times \vec{a})$. To receive full credit, your resultant vector must have a reasonably correct direction AND magnitude. [4 pts]



$|c \times a| = \frac{\sqrt{2}}{2} |\vec{a}|^2$

$|a \times a| = |a|^2 \cdot \frac{\sqrt{2}}{2}$

$|b \times (c \times a)| = |a|^3 \cdot \frac{\sqrt{2}}{2}$

$|\vec{b} \times (\vec{c} \times \vec{a})|$ is assuming $|\vec{a}| = 1$

11. Consider the plane x – 3y + 2z -13 = 0, with a line that is perpendicular to the plane. The line also passes through the point (-2, 1, -1).

a) Write the equation of the line in **parametric** form. [3 pts]

not vector

$\vec{n} = \langle 1, -3, 2 \rangle$

$\vec{r} = -2\hat{i} + \hat{j} - \hat{k} + t(\hat{i} - 3\hat{j} + 2\hat{k})$

b) How far is the point (2, 4, 1) from the plane? Give your answer as a simplified fraction, and rationalize the denominator, if necessary. [3 pts]

$d = \frac{|2 - 12 + 2 - 13|}{\sqrt{14}} = \frac{21\sqrt{14}}{14} = \frac{3\sqrt{14}}{2}$

12. Find the point where the line $< x, y, z > = < -1, 4, 0 > + t < 2,3,1 >$ intersects the plane $x + 2y - 3z = 12$.

[4 pts]

$x = -1 + 2t$

$y = 4 + 3t$

$z = t$

$\frac{x+2y-12}{3} = \frac{2y-3z-13}{2}$

$\frac{3z-x+4}{6} = \frac{x+2y-12}{3}$

$\frac{3z-x+4}{6} = \frac{2y-3z-13}{2}$

$-1 - 2t = -2y + 3z + 12$

$-2t = 3z - 2y + 13$

$t = \frac{2y - 3z - 13}{2}$

$3z - x + 4 = 2x + 4y - 24$

$3x + 4y - 3z = 28$

$2x - 2y + 9z = -15$

$x + 6y - 12z = 43$

$x + 2y - 3t = 12$

$-3t = 12 - x - 2y$

$t = \frac{x + 2y - 12}{3}$

$x + 8 + 6t - 3z = 12$

$6t = 4 + 3z - x$

$t = \frac{3z - x + 4}{6}$

$2x + 4y - 24 = 6y - 9z - 3$

$3z - x + 4 = 6y - 9z - 39$

$3x - 4y - 3z = 28$

$4x - 4y + 18z = -30$

$7x + 15z = -2$

$12x - 2y + 9z = 15$

$2x + 12y - 24z = 86$

$-14y + 33z = 71$

$3x + 4y - 3z = 28$

$3x + 14y - 36z = 129$

$(-1 + 2t) + 2(4 + 3t) - 3(t) = 12$

$-1 + 2t + 8 + 6t - 3t = 12$

$5t = 5, \; t = 1$

$(1, 7, 1)$

13. Find two values of m that make the vectors $< 4,1,-7 >$ and $< m, m^2, 3 >$ orthogonal. [4 pts]

$4m + m^2 - 21 = 0$

$(m-3)(m+7) = 0$

$m = 3$ or -7

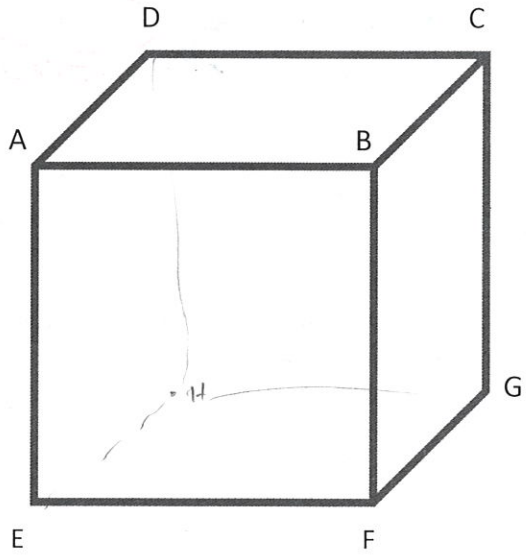
Questions 14 and 15 refer to the diagram on right, which shows cube with vertices labeled A-H (point H is the vertex that is behind the figure, out of sight).

14. Use the vertices to name a vector that has the same direction as the given cross product. Each of your answers should be a single vector, identified by 2 points. [1 pt each]

$\overrightarrow{BA} \times \overrightarrow{BF}$ $\overrightarrow{CB}$

$\overrightarrow{CA} \times \overrightarrow{CD}$ $\overrightarrow{CG}$

$\overrightarrow{EA} \times \overrightarrow{EB}$ $\overrightarrow{EH}$



15. Fill in the blank with a <, >, or = symbol to make the statement true. [1 pt]

$|\overrightarrow{BA} \times \overrightarrow{BF}|$ $=$ $|\overrightarrow{EA} \times \overrightarrow{EB}|$

a^2 $\quad$ $a \cdot a\sqrt{2} \cdot \frac{\sqrt{2}}{2}$

-0