

1. Answer "True" or "False" for each statement. [1 pt each]

- a) If a sequence is always increasing, it must diverge. _____
- b) If a sequence converges, then it must have both an upper and a lower bound. _____
- c) If a sequence is NOT bounded above, then it must be bounded below. _____
- d) If a sequence $\{a_n\}$ is everywhere decreasing, then $a_n > a_{n+1}$ for all values of n . _____
- e) A sequence can converge to two different values at once. _____
- f) If $a_n \leq b_n \leq c_n$ for all n , and a_n and c_n converge to the same value, then b_n also converges. _____
- g) If $a_n < n$ for all n , this proves that $\{a_n\}$ converges. _____

2. Fill in the blanks. [1 pt each]

- a) The sequence $\{p^n\}$ will converge if _____
- b) The sequence $\{n^p\}$ will converge if _____
- c) If $a_n \geq 42$ for all n , this proves that $\{a_n\}$ is _____.

3. For each pair of bolded and underlined words, circling one of the two will make the entire statement true. Circle the correct choice for each pair. [1 pt per circle]

The sequence $\{\cos(n)\}$ (converges / diverges) because in constructing a neighborhood proof, no matter

which limit you use, there will be a(n) (finite / infinite) number of terms (outside / inside) the neighborhood.

3. For each sequence, state the value to which the sequence converges, or state that the sequence diverges. CLEARLY justify your answer with mathematical proof or other work. If your work is correct but is difficult to interpret, you may not receive full credit.

You must state which test or principle you use for each case. Also, you may NOT use the same test/principle for both problems. [5 pts each – 1 pt per blank on the right, 3 pts for clear justification]

a) Sequence: $\left\{ \frac{3n+5}{n+1} \right\}$

Test/Principle used: _____

the value to which the sequence converges (or say “diverges”): _____

Show your work here:

b) Sequence: $b_n = \begin{cases} 10 & \text{if } n \text{ is odd} \\ \frac{20n-1}{2n} & \text{if } n \text{ is even} \end{cases}$

Test/Principle used: _____

the value to which the sequence converges (or say “diverges”): _____

Show your work here: