Analysis Unit4 Vectors and Parametrics Quiz1
Hahn/Hlasek/Tantod
NO Calculator

30
30 points

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Period: 7

1. Given vectors $\vec{a} = \langle -3, 3 \rangle$ and $\vec{b} = \langle 2, -4 \rangle$, find each. Values should be in exact form. [2pts each]

a) $\vec{b} + 2\vec{a} - 5\vec{j}$

$= \boxed{-4\hat{i} - 3\hat{j}}$

b) Find a unit vector that is in the same direction as $\vec{b}$

$\frac{1}{\sqrt{20}}\vec{b}$

$= \frac{\cancel{2}}{\cancel{2}\sqrt{5}}\hat{i} - \frac{4}{2\sqrt{5}}\hat{j} = \boxed{\frac{1}{\sqrt{5}}\hat{i} - \frac{2}{\sqrt{5}}\hat{j}}$

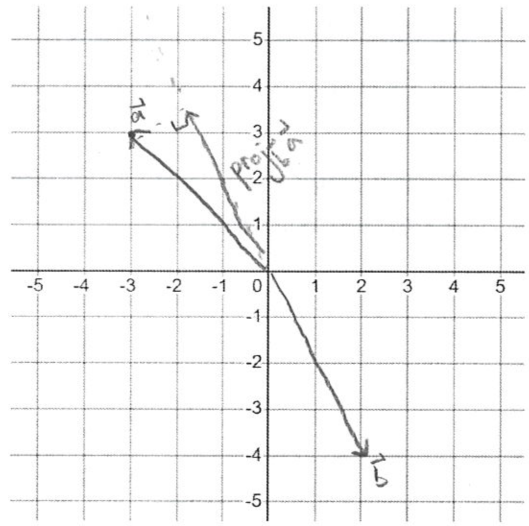
c) Find the vector projection, $proj_{\vec{b}}\vec{a}$

$proj_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$

$= -\frac{18}{20}(2\hat{i} - 4\hat{j})$

$= \boxed{-\frac{9}{5}\hat{i} + \frac{18}{5}\hat{j}}$

d) On the axis below, sketch and label $\vec{a}$ and $\vec{b}$ in standard position. Then sketch vector projection, $proj_{\vec{b}}\vec{a}$. Label each vector so they can be clearly distinguished.



2. Consider the points A= (-2, 5, 1) and B = (4, 8, -2) and C (8, y, 3)

a) Find the vector equation of the line containing A and B [3 pts]

$\boxed{-2\hat{i} + 5\hat{j} + \hat{k} + t(6\hat{i} + 3\hat{j} - 3\hat{k})}$

b) Find 2 points on the line that are 3 times as far from A as they are from B. [3 pts]

$(\frac{5}{2}, \frac{29}{4}, -\frac{5}{4}); (7, \frac{19}{2}, -\frac{7}{2})$

$x + 1 = 3x$
$x = \frac{1}{2}$

c) Solve for the y component of point C so that $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are orthogonal. [3 pts]

$\vec{AB} = \langle 6, 3, -3 \rangle$, $\vec{AC} = \langle 10, y-5, 2 \rangle$

$\vec{AB} \cdot \vec{AC} = 0$
$54 + 3(y-5) = 0$
$3y = -39$
$y = 13$

$\boxed{y = -13}$

-0

3. Given the following set of parametric equations:

$x(t) = \sqrt{3t - 1} \qquad y(t) = \frac{3}{t}$

a) Eliminate the parameter and write an equation as a function y in terms of x. [3pts]

$x = \sqrt{3t-1}$

$x^2 = 3t - 1$

$t = \frac{x^2+1}{3}$

$y = \frac{3}{t} = \frac{3}{\frac{x^2+1}{3}} = \frac{9}{x^2+1}$

$\boxed{y(x) = \frac{9}{x^2+1}}$

b) State the domain and range of the graph. [2 pts]

$x \in [0, \infty), \; y \in (0, 9]$

4. For the following set of parametric equations, eliminate the parameter. Then name the type of graph that is formed.

$x(t) = 2 + 4sect \qquad t: [0, 2\pi]$
$y(t) = 3 + 2tant$

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$x = 2 + 4\sec\left(\tan^{-1}\left(\frac{y-3}{2}\right)\right)$

$y - 3 = 2\tan t$

$t = \arctan\left(\frac{y-3}{2}\right)$

$(x-2)^2 = 16\sec^2 t = 16\left[\left(\frac{y-3}{2}\right)^2 + 1\right]$

$(x-2)^2 - 4(y-3)^2 = 16$

Equation: $(x-2)^2 - 4(y-3)^2 = 16$ [3 pts] Type of graph: hyperbola [1 pt]

5. A stomp rocket is launched from the ground with a velocity of 60 ft/s at an angle of 30 degrees to the horizontal, and flies towards a 9-foot wall that is at a horizontal distance of 30 ft from the point of launch. Will the stomp rocket hit the wall or go over the wall? You must justify your answer to receive credit. [4]

$x = (v_0 \cos\theta)t$

$t = \frac{x}{v_0 \cos\theta}$

$\frac{30}{\sqrt{3}} > 14\frac{1}{3}$

$y = t v_0 \sin\theta - 16t^2$

$y = x\tan\theta - \frac{16x^2}{v_0^2\cos^2\theta}$

$= (30 ft) \cdot \frac{1}{\sqrt{3}} - \frac{(16 \frac{ft}{s^2})(\cancel{900} ft^2)}{(\cancel{3600} \, ft^2/s^2) \cdot \frac{3}{\cancel{4}}}$

$= \left(\frac{30}{\sqrt{3}} - \frac{16}{3}\right) ft > 9 ft \Rightarrow$ $\boxed{\text{Rocket makes it over the wall}}$ ✓

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