28/30

Group Theory Quiz #1
Analysis 2022-23
Hahn/Hlasek/Tantod [30 points]

I'm flipping out over this quiz!!! Justin Oh

1. What are the requirements a set of elements must satisfy in order for them to be a group under a certain operation? [3]

closure
identity
invertibility
associativity

2. Consider the four-member collection of elements Q, R, S and T as displayed in the table below under the operation @

	Q	R	S	T
Q	Q	R	S	T
R	R	S	T	Q
S	S	T	Q	R
T	T	Q	R	S

$T^2 = S$
$T^3 = R$
$T^4 = Q$

a) Is there an identity in this group? If so, name it and defend your answer. If not, justify. [2]

Q, the composition of Q and any other operation, returns the same operation.

b) Does every element have an inverse element? Name the inverse of each element that has one.[2]

Yes: $Q^{-1} = Q$
$R^{-1} = T$
$S^{-1} = S$
$T^{-1} = R$

c) Name the period of element T (or state that it does not have one) [2]

element T has a period of 4.

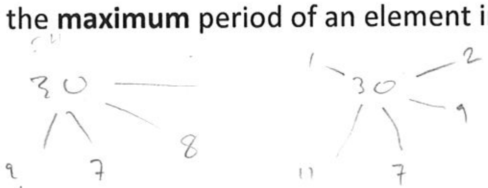
d) Do the set of these elements satisfy the requirements to be a group? [2]

yes.

3. Draw a 10-post element that would have a period of 16 or state there is no such element. [2]

No such element.

-2 4. What is the **maximum** period of an element in the 30-post snap group? Justify your reasoning.[3]



lcm(1, 2, 9, 7, 11) = 198 × 7 = 1386

1386, because it is the lcm of a group of numbers which add up to 30 and are prime, relative to one another.

period can be much bigger

F2

5. Is the set of numbers: { 1 , -1, i, -i} a group under multiplication? If not, justify using the requirements to be a group. [2]

yes.

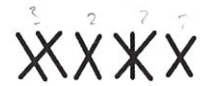
6. Is the set of numbers: { 1 , -1, i, -i} a group under addition? If not, justify using the requirements to be a group. [2]

No, 1+(-1) is 0 which is not in the group
⇒ does not satisfy closure
⇒ {1, -1, i, -i} is not a group under addition

7. Below are some of the elements of the 10-post snap group. Below each element, write down its period. [4]

a) 

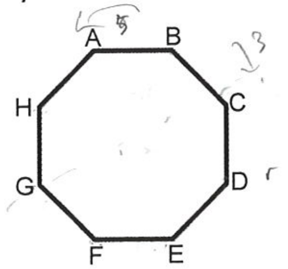
period: 10

b) 

period: 6

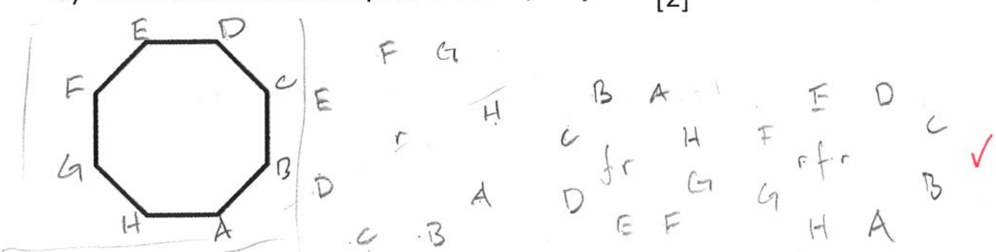
8. Consider the dihedral (reflection/rotation) group for the regular octagon where the element shown is the identity element.

$\frac{360}{8} = 45$ $\frac{225}{45} = 5$



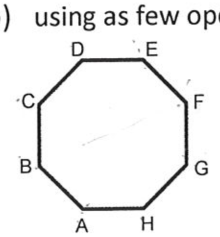
Let "f" be defined as the operation "reflect over the GC line in the identity. Let "r" be defined as the operation rotate counterclockwise by 225 degrees.

a) Draw the element represented by $r \cdot f \cdot r$ [2]



✓

b) using as few operations as possible, name the element below (use r's and f's). [3]



f rf

c) If we just use r, rotate ~~clockwise~~ counterclockwise by 225 degrees, as a generator, can we generate all the elements of the dihedral group for the regular octagon? [1]

No, the dihedral group of 8 contains 8 "mirrored states" which require a reflection to achieve.