Analysis H
Hahn/ Hlasek/ Tantod
Unit 1: AtPS, Quiz 1
NO CALCULATORS

26 26.5
27 points

We have a quiz today??? **Series-ly** Justin Oh
Period 7

Odd-Number Triangle- (reminder: in the Odd# Triangle, the row with [3 5] is the 2nd row)

1. Write "true" or "false" for each statement. (1 pt each)

   a) In the odd-numbered triangle, the difference between the last term of the nth row and the first term of the nth row is $2n-1$. false

   1
   3 5
   7 9 11

   b) The sum of the terms in the nth row of the odd-numbered triangle is n^3. true

   △ c) The sum of the first k cube numbers is a square number. true

   = sum of consecutive odds $= \frac{(2n-1)+1}{2} \cdot n = n^2$

Sequences and Series

2. Find the sum of each expression. Since you don't have a calculator, no need to calculate the actual number. Just give an equivalent numerical expression for each answer. (3 pts each)

   a) $13 + 16 + 19 + 22 + \ldots + 103 =$ $\frac{13+103}{2} \cdot n =$ $\boxed{\frac{13+103}{2} \cdot 31}$

   +3 +3 +3

   $\frac{103-13}{3} + 1 = \frac{90}{3} + 1 = 31 = n$

   b) Find the sum of the first 10 terms of the following series: $\overset{2^8}{256} + \overset{2^6 \cdot 3}{192} + \overset{2^4 \cdot 3^2}{144} + \overset{2^2 \cdot 3^3}{108} + \overset{3^4}{81} + \ldots$

   $S_{10} = 256 + 256 \cdot r + 256 \cdot r^2 + \cdots + 256 r^9$ $a_1 = 256,\ r = \frac{3}{2^2}$

   $-\left|\ \frac{3S_{10}}{4} = 256r + 256r^2 + \cdots + 256r^9 + 256r^{10}\right.$

   $S_{10}\left(1-\frac{3}{4}\right) = 256 - 256r^{10} = 256\left(1-\left(\frac{3}{4}\right)^{10}\right)$, $S_{10} = 256\left(1-\left(\frac{3}{4}\right)^{10}\right) \cdot 4 = \boxed{1024\left(1-\left(\frac{3}{4}\right)^{10}\right)}$

Pascal's Triangle

3. The last 5 terms of the 16th row of Pascal's triangle are: 1820, 560, 120, 16, and 1.
   What are the last 5 digits of 11^{16}? (2 points)

   $\boxed{11^{16} = \ldots 72161}$

   1
   16
   120
   560
   + 1820
   72161

   1
   1 1
   1 2 1
   1 3 3 1
   1 4 6 4 1
   1 5 10 10 5 1
   161051

4. Simplify each as a single term, or single binomial coefficient. (2pts each)

   a) $\binom{15}{1} + \binom{15}{3} + \binom{15}{5} + \binom{15}{7} + \cdots + \binom{15}{15} = 2^{14}$

   b) $\binom{42}{0} + \binom{43}{1} + \binom{44}{2} + \binom{45}{3} + \cdots + \binom{70}{28} = \binom{71}{28}$

   c) $\binom{50}{6} + 2\binom{50}{7} + \binom{50}{8} = \binom{52}{8}$

-0

5. The first 5 rows of triangular pattern is shown below, where all terms are multiples of 3. For reference, the bolded "24" is a_8, and is the 2nd term of the 4th row, and is also the 3rd term of the 2nd column. The bolded "42" is a_{14}, which is the 4th term of the 5th row, and the 2nd term of the 4th column.

3				
6	9			
12	15	18		
21	**24**	27	30	
33	36	39	**42**	45

a) In which row is a_{201}? (2pts)

$C_n = \frac{n(n+1)}{2}$ = number of terms in n rows

$201 \le \frac{n(n+1)}{2}$, $\frac{20\cdot 21}{2} = 210$, $\frac{20\cdot 19}{2} = 190$ $\Rightarrow$ $\boxed{a_{201} \text{ is in row } 20}$

−1 b) What is the 8th term of the 2nd column. (1pt)

= 2nd term of the 9th row

$C_8 = \frac{8(9)}{2} = 36$ $\boxed{a_{38} = 3\cdot 38}$ = 114

2nd term of the ninth row is $C_8 + 2 = 38$,

c) Find an expression for the nth term of the 2nd column. (3pts)

$a(C_n + 2)$

$= \boxed{3\left[\frac{(n+1)n}{2} + 2\right]}$

Fibonacci Numbers

5. $F_{158} = F_a F_b + F_{84} F_{73}$. Find a and b (1 pt)

$84 + 73 = 157$

$\boxed{a = 85 \\ b = 74}$

m | n

$F_{m+n+1} = F_m F_n + F_{m+1} F_{n+1}$

6. Find a compact expression for: $F_0 - F_1 + F_2 - F_3 + F_4 - \cdots - F_{2n-1} + F_{2n}$ (3 pts)

0 1 1 2 3

$\cancel{F_0} - F_1 + F_2 - F_3 \cdots - F_{2n-1} + F_{2n}$

$-F_1 + \cancel{F_3} - F_1$

$-F_3 + \cancel{F_5} - \cancel{F_3}$

$-F_5 + \cancel{F_7} - \cancel{F_5}$

$\vdots$

$-F_{2n-1} + F_{2n+1} - \cancel{F_{2n-1}}$

$-2F_1 - F_3 - F_5 - \cdots - F_{2n-1} + F_{2n+1}$

$= -2F_1 - F_3 - F_5 - \cdots - F_{2n-3} + F_{2n}$

$= -2F_1 - F_3 - F_5 - \cdots - F_{2n-3} + F_{2n-2} + F_{2n-1}$

$= -2F_1 - F_3 - F_5 - \cdots - F_{2n-5} + F_{2n-4} + F_{2n-1}$

$= -2F_1 - F_3 - F_5 - \cdots - F_{2n-7} + F_{2n-6} + F_{2n-1}$

$\vdots$

$= -2F_1 - F_3 + F_4 + F_{2n-1}$

$= -2F_1 + F_2 + F_{2n-1} = \boxed{F_{2n-1} - 1}$

F1