Analysis H – Hahn / Hlasek / Tantod 20.5 Justin Oh went to the DMV to get a diverge license
Unit 8: Limits, Quiz 1 on Sequences Period: 7
No Calculators 23 points

1. Answer "True" or "False" for each statement. [1 pt each]

a) If a sequence is always increasing, it must diverge. False

b) If a sequence converges, then it must have both an upper and a lower bound. True

c) If a sequence is NOT bounded above, then it must be bounded below. False

-1 d) If a sequence $\{a_n\}$ is everywhere decreasing, then $a_n > a_{n+1}$ for all values of n. ~~True~~

e) A sequence can converge to two different values at once. False

f) If $a_n \leq b_n \leq c_n$ for all n, and a_n and c_n converge to the same value, then b_n also converges. True

g) If $a_n < n$ for all n, this proves that $\{a_n\}$ converges. False

2. Fill in the blanks. [1 pt each]

-0.5 a) The sequence $\{p^n\}$ will converge if $|p|<1$

b) The sequence $\{n^p\}$ will converge if $p \leq 0$

c) If $a_n \geq 42$ for all n, this proves that $\{a_n\}$ is bounded below.

3. For each pair of bolded and underlined words, circling one of the two will make the entire statement true. Circle the correct choice for each pair. [1 pt per circle]

The sequence $\{\cos(n)\}$ **<u>(converges / diverges)</u>** because in constructing a neighborhood proof, no matter which limit you use, there will be a(n) **<u>(finite / infinite)</u>** number of terms **<u>(outside / inside)</u>** the neighborhood.

-1.5

3. For each sequence, state the value to which the sequence converges, or state that the sequence diverges. CLEARLY justify your answer with mathematical proof or other work. If your work is correct but is difficult to interpret, you may not receive full credit.

You must state which test or principle you use for each case. Also, you may NOT use the same test/principle for both problems. [5 pts each – 1 pt per blank on the right, 3 pts for clear justification]

–1 a) Sequence: $\left\{\frac{3n+5}{n+1}\right\}$ Test/Principle used: ~~Everywhere~~ Decreasing & Boundedness below

the **value** to which the sequence converges (or say "diverges"): 3

Show your work here:

$a_n = \frac{3n+5}{n+1}$

$a_{n+1} = \frac{3(n+1)+5}{n+2} = \frac{3n+8}{n+2}$

$a_{n+1} \leq a_n$

$\frac{3n+8}{n+2} \leq \frac{3n+5}{n+1}$

$\cancel{3n^2}+11n+8 \leq \cancel{3n^2}+11n+10$

$8 \leq 10$

$\Rightarrow a_n$ is non-increasing

$a_n \geq 3$

$\frac{3n+5}{n+1} \geq 3\left(\frac{n+1}{n+1}\right)$

$\frac{3n+5}{\cancel{n+1}} \geq \frac{3n+3}{\cancel{n+1}}$

$5 \geq 3$

$\Rightarrow a_n$ is bound below

a_n converges ($\because a_n$ is non-increasing & bound below)

How do we know it converges to 3?

b) Sequence: $b_n = \begin{cases} 10 & if\ \ n\ is\ odd \\ \frac{20n-1}{2n} & if\ \ n\ is\ even \end{cases}$ Test/Principle used: neighborhood

the **value** to which the sequence converges (or say "diverges"): 10

Show your work here:

$L = 10$

$|b_n - L| = 10 - 10 = 0$ for n odd

$\Rightarrow \forall \varepsilon > 0, |b_n - L| = 0 \leq \varepsilon$ when n is odd

$|b_n - L| = 10 - \frac{20n-1}{2n}$ for n is even

$= \frac{20n}{2n} - \frac{20n-1}{2n}$

$= \frac{1}{2n}$

Let $|b_n - L| = \frac{1}{2n} \leq \varepsilon$

$2n \geq \frac{1}{\varepsilon}$

$n \geq \frac{1}{2\varepsilon}$

$M = \left[\frac{1}{2\varepsilon}\right] + 1 \Rightarrow \forall \varepsilon > 0\ \exists M$ s.t. for $n \geq M: |a_n - L| < \varepsilon$

Since b_n satisfies the neighborhood definition of convergence for both even & odd n around a limit of 10, b_n converges to 10 by the neighborhood principle.

–1