

40 pts

For Questions 1-4, two sets are given, along with three symbols: “greater than”, “less than”, and “equal”. For each question, CIRCLE THE ONE SYMBOL that accurately describes the sets’ relative sizes. [1 pt each]

1. {positive and negative multiples of 7}      > or < or =      {rational numbers greater than 10}
2. {points in 3D space}      > or < or =      {points on a circle}
3. {rotations of an octagon}      > or < or =      {elements in the dihedral group of a pentagon}
4. {prime numbers}      > or < or =      {linear arrangements of all the people on earth}

For Questions 5-7, consider Group P, which is the group of integer powers of 3 under multiplication.

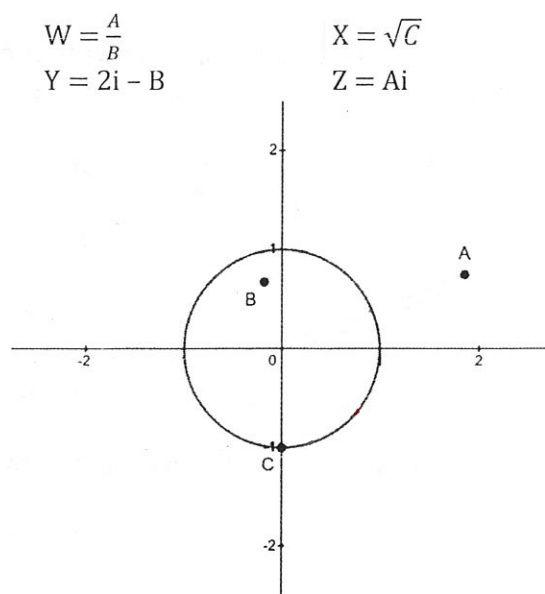
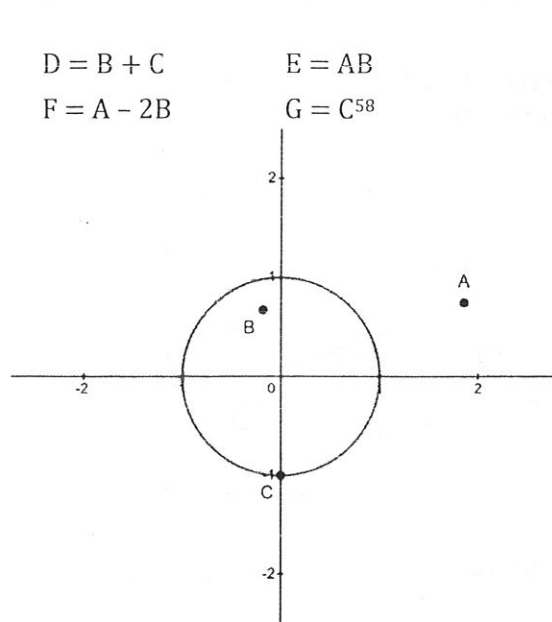
5. MULTIPLE CHOICE (circle the one best answer): The cardinality of Group P is \_\_\_\_\_ [2 pts]  
a) Finite      b) Countably Infinite      c) Uncountably Infinite      d) Not enough information to tell
6. Which element in Group P is the Identity element? \_\_\_\_\_ [1 pt]
7. Group P is isomorphic to \_\_\_\_\_. (circle ALL that apply): [2 pts]  
a) rational numbers under addition      b) rational powers of 2 under multiplication  
c) even integers under addition      d) rotation by an integer number of degrees  
e) rotation by an integer number of radians      f) 3-post snap group

For Questions 8-11, consider Group J, which is the group of  $\left\{ \text{cis } \frac{\pi}{10}, \text{cis } \frac{2\pi}{10}, \dots, \text{cis } \frac{20\pi}{10} \right\}$  under multiplication.

8. Which element in Group J is the Identity element? \_\_\_\_\_ [1 pt]
9. Name an element in Group J that is the inverse of  $\text{cis } \frac{2\pi}{10}$  \_\_\_\_\_ [1 pt]
10. Name 2 different elements in Group J that each have a period of 5. \_\_\_\_\_ and \_\_\_\_\_ [2 pts]
11. Group J is isomorphic to \_\_\_\_\_. (circle ALL that apply): [2 pts]  
a) The rotation group of a 10-gon      b) The rotation group of a 20-gon  
c) The dihedral group of a 10-gon      d)  $\left\{ \frac{k\pi}{10} \right\}$ , where  $k$  is an integer, under addition  
e) The 10-post snap group      f) rotation group of a pyramid with a 20-gon base

12. The diagrams each show a **circle of radius 1**, along with complex numbers A, B, and C.

Graph and label the complex numbers (D, E, F, and G on the left, and W, X, Y, and Z on the right). [2 pts each]



13. Given:  $(\cos\theta)^4 = \cos^4\theta + 4(\cos^3\theta)(i\sin\theta) + 6(\cos^2\theta)(i^2\sin^2\theta) + 4(\cos\theta)(i^3\sin^3\theta) + i^4\sin^4\theta$   
 Use the expansion above, along with DeMoivre's Theorem, to express  $\cos 4\theta$  in terms of  $\sin\theta$  and  $\cos\theta$ .  
 [2 pts]

14. Given:  $|z| = 2$  and  $\text{Arg}(z) = \theta$ . Express  $\text{Arg}(z(z+2))$  in terms of  $\theta$ . [3 pts]

15. Write a 2x2 matrix that will perform the given transformation. [1 pt each]

a) Dilation by a factor of 5

b) Reflection over the y-axis

c) Rotation by 180 degrees

d) Map all points to the line  $y = 3x$   
 (but your answer cannot be a zero matrix)