Analysis H 2022-2023
Hahn / Hlasek/ Tantod
Unit 8: Sequences and Series Quiz 2
No Calculators!

Why so series? ____________________
Period: ______

____
54 points

#1-4 are Multiple Choice: Circle the best answer. [3 pts each]

1. Which of these is NOT a test for convergence for sequences?

a) neighborhood test
b) always increasing/decreasing and bounded above/below
c) n^{th}term test
d) domination principle
e) none of the above (all the listed answers can be used to test for convergence)

2. The sequence $\left\{\frac{\sqrt{10n^3+2n^2+4n^4+1}}{\sqrt{8n^2-14n^3+2n^4-n-5}}\right\}$ converges to __________

a) $\frac{\sqrt{5}}{2}$ b) $\sqrt{2}$ c) $\frac{1}{2}$ d) 0 e) $\frac{5\sqrt{7}}{7}$

3. For what values of n is the sequence $a_n = 5n - n^2 - 1$ decreasing?

a) $n \geq 3$ b) $n \leq 3$ c) $n \geq 2$ d) $n \geq 1$ e) $n \leq 2$

4. $\sum 7r^n$ will diverge for what values of r?

a) $r > 0$ b) $r \geq 1$ c) $|r| \geq 1$ d) $r < 1$ e) $|r| \leq 1$

5. For each statement, answer True or False [1 pt each]

a) A certain sequence converges to $\frac{1}{2}$. Its corresponding series will diverge.________________

b) A certain sequence converges to $\frac{1}{2}$. There will be an infinite number of terms outside of any neighborhood around $\frac{1}{2}$.________________

c) If a series converges to a value, its sequence of partial sums will converge to the same value.________________

d) For sequence $\{a_n\}$ if the limit of $\frac{a_{n+1}}{a_n}$ approaches $\frac{3}{2}$, then the corresponding series $\sum_{n=1}^{\infty} a_n$ will converge________________

e) If a sequence $a_n$converges to 0, then the series $\sum a_n$ converges________________

6. Consider a sequence $\{t_n\}$ and its corresponding series $\sum_{n=1}^{\infty} t_n$

For all questions name a sequence that satisfies the given requirements, or state that none exists. [2pts each]

a) $\{t_n\}$ converges to zero and $\sum_{n=1}^{\infty} t_n$ diverges. $\{t_n\} =$ ______________

b) $\{t_n\}$ converges to zero and $\sum_{n=1}^{\infty} t_n$ converges. $\{t_n\} =$ ______________

c) $\{t_n\}$ alternates and $\sum_{n=1}^{\infty} t_n$ converges to 3. $\{t_n\} =$ ______________

7. Given three sequences $\{r_n\}, \{s_n\}, \{t_n\}$ such that $\{r_n\} \leq \{s_n\} \leq \{t_n\}$ for all n. What must be true about $\{r_n\}$ and $\{t_n\}$ to conclude that $\lim_{n\to\infty} s_n = 5$ [2 pts]

8. Write a series that converges to $\frac{5}{3}$. Give your answer in Sigma notation. [4 pts]

9. For each sequence, write "C" if it converges and "D" if it diverges [1 pt each]

a) $\left\{\frac{2}{7n^2}\right\}$________ b) $\left\{\frac{n^2+1}{\cos^2 n}\right\}$________ c) $\left\{\frac{5}{\sqrt[5]{n}}\right\}$ ________ d) $\left\{\frac{n+1}{n\sqrt{n}}\right\}$________

10. Justify your answer to 9(b) by using one of the tests we learned in class. In your answer, include the name (or explanation) of the test. [3pts]

11. Justify your answer to 9(c) by using one of the tests we learned in class. In your answer, include the name (or explanation) of the test. [3pts]

Questions 12-14: [5 pts each]

For each series, write a clear proof to show convergence or divergence, first indicating the name of the test you used. **Important: for these 3 problems, you MAY NOT use the same test twice!** If you use the same test for more than one problem, you will lose 3 points per infraction. Do the work for #12 and 13 on this page, and then the work for #14 on the back of this page.

12. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n+1}$ Test used: ______________________________

13. $\sum_{n=1}^{\infty} \frac{n!\,3^{2n}}{4^n(n+3)!}$ Test used: ______________________________

14. $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$ (The series is written here so you can see it and plan which test you want to do, but do your work for #14 on the back of this page)

14. $$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$$

Test used: ____________________________________