Analysis H 2022-2023
Hahn / Hlasek/ Tantod
Unit 8: Sequences and Series Quiz 2
No Calculators!

Why so series? Justin Oh
Period: 7

50
54 points

#1-4 are Multiple Choice: Circle the best answer. [3 pts each]

1. Which of these is NOT a test for convergence for sequences?

a) neighborhood test
b) always increasing/decreasing and bounded above/below
c) n^{th}term test
d) domination principle
e) none of the above (all the listed answers can be used to test for convergence)

2. The sequence $\left\{\frac{\sqrt{10n^3+2n^2+4n^4+1}}{\sqrt{8n^2-14n^3+2n^4-n-5}}\right\}$ converges to __________

$\frac{\sqrt{4}}{\sqrt{2}} = \sqrt{2}$

a) $\frac{\sqrt{5}}{2}$
b) $\sqrt{2}$
c) $\frac{1}{2}$
d) 0
e) $\frac{5\sqrt{7}}{7}$

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3. For what values of n is the sequence $a_n = 5n - n^2 - 1$ decreasing?

$a_n \geq a_{n+1}$

a) $n \geq 3$
b) $n \leq 3$
c) $n \geq 2$
d) $n \geq 1$
e) $n \leq 2$

$a_1 = 3,\ a_2 = 5,\ a_3 = 5$

4. $\sum 7r^n$ will diverge for what values of r?

a) $r > 0$
b) $r \geq 1$
c) $|r| \geq 1$
d) $r < 1$
e) $|r| \leq 1$

5. For each statement, answer True or False [1 pt each]

a) A certain sequence converges to $\frac{1}{2}$. Its corresponding series will diverge. True

b) A certain sequence converges to $\frac{1}{2}$. There will be an infinite number of terms outside of any neighborhood around $\frac{1}{2}$. False

c) If a series converges to a value, its sequence of partial sums will converge to the same value. True

d) For sequence $\{a_n\}$ if the limit of $\frac{a_{n+1}}{a_n}$ approaches $\frac{3}{2}$, then the corresponding series $\sum_{n=1}^{\infty} a_n$ will converge False

e) If a sequence $a_n$converges to 0, then the series $\sum a_n$ converges False

-3

6. Consider a sequence $\{t_n\}$ and its corresponding series $\sum_{n=1}^{\infty} t_n$

For all questions name a sequence that satisfies the given requirements, or state that none exists. [2pts each]

a) $\{t_n\}$ converges to zero and $\sum_{n=1}^{\infty} t_n$ diverges. $\{t_n\} =$ $\frac{1}{n}$ $S_\infty = \frac{a_1}{1-r}$

b) $\{t_n\}$ converges to zero and $\sum_{n=1}^{\infty} t_n$ converges. $\{t_n\} =$ $\frac{1}{n^2}$ $\frac{a_1}{4/3} = 3$

-1 c) $\{t_n\}$ alternates and $\sum_{n=1}^{\infty} t_n$ converges to 3. $\{t_n\} =$ $4\left(-\frac{1}{3}\right)^{n-1}$ $a_1 = 4$

7. Given three sequences $\{r_n\}, \{s_n\}, \{t_n\}$ such that $\{r_n\} \leq \{s_n\} \leq \{t_n\}$ for all n. What must be true about $\{r_n\}$ and $\{t_n\}$ to conclude that $\lim_{n\to\infty} s_n = 5$ [2 pts]

$\{r_n\} = 0$
$\{s_n\} = \frac{1}{n^2}$
$\{t_n\} = \frac{1}{n}$

to conclude that $\lim_{n\to\infty} s_n = 5$, both $\{r_n\}$ and $\{t_n\}$ must both converge to 5 as n approaches ∞, in addition to $\{r_n\} \leq \{s_n\} \leq \{t_n\}$

8. Write a series that converges to $\frac{5}{3}$. Give your answer in Sigma notation. [4 pts]

$\frac{a_1}{1-r} = \frac{5}{3}$

$\frac{3}{5} = 1 - r$; $r = 2/5$

$$\boxed{\sum_{n=0}^{\infty} \left(\frac{2}{5}\right)^n = \frac{5}{3}}$$

9. For each sequence, write "C" if it converges and "D" if it diverges [1 pt each]

a) $\left\{\frac{2}{7n^2}\right\}$ C b) $\left\{\frac{n^2+1}{\cos^2 n}\right\}$ D c) $\left\{\frac{5}{\sqrt[5]{n}}\right\}$ C d) $\left\{\frac{n+1}{n\sqrt{n}}\right\}$ C

10. Justify your answer to 9(b) by using one of the tests we learned in class. In your answer, include the name (or explanation) of the test. [3pts]

since cos has a range of $[-1, 1]$, $\cos^2$ has a range of $[0, 1]$

$\Rightarrow \cos^2 \leq 1$

$\Rightarrow \frac{n^2+1}{\cos^2 n} \geq \frac{n^2+1}{1}$

$n^2+1 > n$

$n^2 - n > -1$ for $n > 0$

since $\frac{n^2+1}{\cos^2 n} > n^2+1 > n$

$\Rightarrow \frac{n^2+1}{\cos^2 n} > n$

$\Rightarrow \frac{n^2+1}{\cos^2 n}$ diverges by comparison to $\{n\}$, a divergent sequence.

✓

11. Justify your answer to 9(c) by using one of the tests we learned in class. In your answer, include the name (or explanation) of the test. [3pts]

$c_n = \frac{5}{\sqrt[5]{n}} = 5 \cdot \frac{1}{n^{1/5}}$

$\left\{\frac{1}{n^{1/5}}\right\}$ converges ($\because$ it is of the form $\frac{1}{n^p}$, where $p = 1/5 > 0$)

$\Rightarrow \{c_n\}$ converges ($\because$ c_n is an integer multiple of a convergent sequence)

✓

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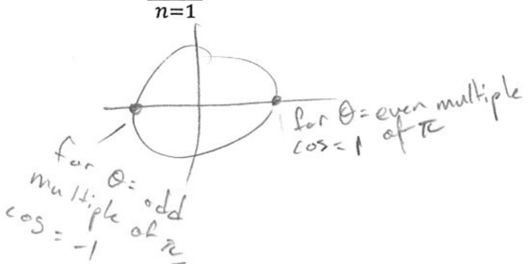
Questions 12-14: [5 pts each]

For each series, write a clear proof to show convergence or divergence, first indicating the name of the test you used. **Important: for these 3 problems, you MAY NOT use the same test twice!** If you use the same test for more than one problem, you will lose 3 points per infraction. Do the work for #12 and 13 on this page, and then the work for #14 on the back of this page.

12. $$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n+1}$$

Test used: Alternating Series Test



$\Rightarrow \cos(n\pi) = (-1)^n$ for integer n

($\because (-1)^n = 1$ for even n and $(-1)^n = -1$ for odd n)

$\Rightarrow a_n = \frac{\cos(n\pi)}{n+1} = \frac{(-1)^n}{n+1}$

Since $(-1)^n$ alternates between -1 & 1, and $\frac{1}{n+1} > 0$ (for $n > 0$) $\frac{(-1)^n}{n+1}$ is an alternating sequence

$|a_n| \geq |a_{n+1}|$

$\left|\frac{(-1)^n}{n+1}\right| \geq \left|\frac{(-1)^n}{(n+1)+1}\right|$

$\frac{1}{n+1} \geq \frac{1}{n+2}$

$\cancel{n}+1 \leq \cancel{n}+2$

$1 \leq 2$

$\Rightarrow |a_n|$ is everywhere decreasing

$\lim_{n\to\infty} \frac{(-1)^n}{n+1} = \lim_{n\to\infty} \left|\frac{(-1)^n}{n+1}\right| = \lim_{n\to\infty} \frac{1}{n+1} = 0$

($\because |0| = 0$)

Since $a_n = \frac{\cos(n\pi)}{n+1} = \frac{(-1)^n}{n+1}$ & a_n alternates, $|a_n|$ is everywhere decreasing, and $\lim_{n\to\infty} a_n = 0$, $\sum a_n$ converges by the Alternating Series Test ✓

13. $$\sum_{n=1}^{\infty} \frac{n!\,3^{2n}}{4^n(n+3)!}$$

Test used: Ratio Test

$a_n = \frac{n!\,3^{2n}}{4^n(n+3)!}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$

$= \lim_{n\to\infty} \dfrac{\frac{(n+1)\cancel{n!}\cdot 3^2\cdot \cancel{3^{2n}}}{4\cdot\cancel{4^n}(n+4)\cancel{(n+3)!}}}{\frac{\cancel{n!}\cdot\cancel{3^{2n}}}{\cancel{4^n}\cancel{(n+3)!}}}$

$= \lim_{n\to\infty} \frac{9(n+1)}{4(n+4)}$

$= \frac{9}{4} > 1$

$\Rightarrow \boxed{\sum a_n \text{ diverges by the Ratio Test}}$ ✓

14. $$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$$

(The series is written here so you can see it and plan which test you want to do, but do your work for #14 on the back of this page)

14. $$\sum_{n=1}^{\infty} \frac{\ln(n)}{n^3}$$

Test used: Direct Comparison Test.

$a_n = \frac{\ln(n)}{n^3}$

$\ln(n) < n$

$n < e^n$ for $n > 1$

$\Rightarrow \frac{\ln(n)}{n^3} < \frac{n}{n^3} = \frac{1}{n^2}$ for $n > 0$

n^3 and $\ln(n)$ are both non-negative for $n > 0$

$\Rightarrow 0 \le \frac{\ln(n)}{n^3} \le \frac{1}{n^2}$ for $n > 0$

Since $\sum \frac{1}{n^2}$ converges as a p-series

$\Rightarrow \sum \frac{\ln(n)}{n^3}$ converges by the Direct Comparison Test.

✓