Analysis H 2023 – Hahn / Hlasek / Tantod
GAtM quiz 2

39
40 pts

Justin Oh is having uncountably infinite fun
Period: 7

For Questions 1-4, two sets are given, along with three symbols: "greater than", "less than", and "equal". For each question, CIRCLE THE ONE SYMBOL that accurately describes the sets' relative sizes. [1 pt each]

1. {positive and negative multiples of 7} **>** or **<** or **(=)** {rational numbers greater than 10}
2. {points in 3D space} **>** or **<** or **(=)** {points on a circle}
3. {rotations of an octagon} **>** or **(<)** or **=** {elements in the dihedral group of a pentagon}
4. {prime numbers} **(>)** or **<** or **=** {linear arrangements of all the people on earth}

For Questions 5-7, consider Group P, which is the group of integer powers of 3 under multiplication.

5. MULTIPLE CHOICE (circle the one best answer): The cardinality of Group P is _____ [2 pts]

   a) Finite (b) Countably Infinite c) Uncountably Infinite d) Not enough information to tell

6. Which element in Group P is the Identity element? $3^0 = 1$ [1 pt]

7. Group P is isomorphic to _____. (circle ALL that apply): [2 pts]

   a) rational numbers under addition
   b) rational powers of 2 under multiplication
   (c) even integers under addition
   d) rotation by an integer number of degrees
   (e) rotation by an integer number of radians
   f) 3-post snap group

For Questions 8-11, consider Group J, which is the group of $\left\{cis\frac{\pi}{10}, cis\frac{2\pi}{10}, \ldots, cis\frac{20\pi}{10}\right\}$ under multiplication.

8. Which element in Group J is the Identity element? $cis\frac{20\pi}{10}$ [1 pt]

9. Name an element in Group J that is the inverse of $cis\frac{2\pi}{10}$ $cis\frac{18\pi}{10}$ [1 pt]

10. Name 2 different elements in Group J that each have a period of 5. $cis\frac{4\pi}{10}$ and $cis\frac{6\pi}{10}$ [2 pts]

11. Group J is isomorphic to _____. (circle ALL that apply): [2 pts]

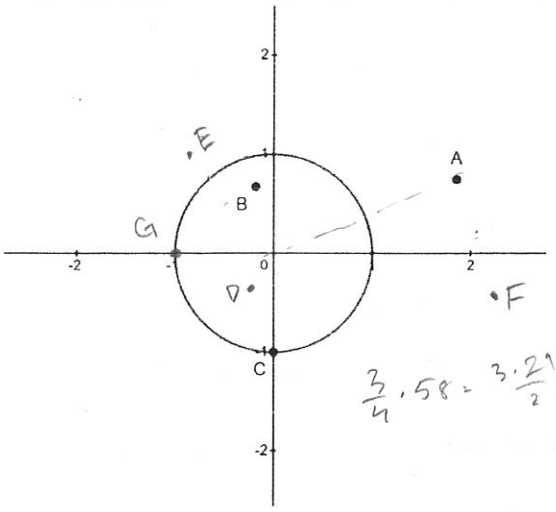
   a) The rotation group of a 10-gon
   (b) The rotation group of a 20-gon
   c) The dihedral group of a 10-gon
   d) $\left\{\frac{k\pi}{10}\right\}$, where k is an integer, under addition
   e) The 10-post snap group
   (f) rotation group of a pyramid with a 20-gon base

-0

12. The diagrams each show a **circle of radius 1**, along with complex numbers A, B, and C.
Graph and label the complex numbers (D, E, F, and G on the left, and W, X, Y, and Z on the right). [2 pts each]

$D = B + C$ $\quad E = AB$

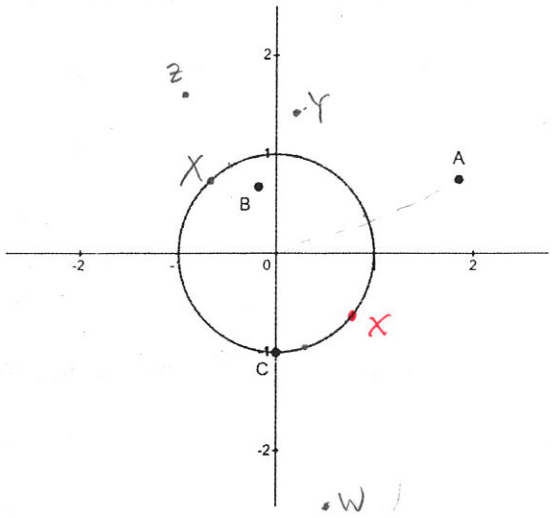
$F = A - 2B$ $\quad G = C^{58}$



$\frac{3}{4} \cdot 58 = 3 \cdot \frac{29}{2}$

$W = \frac{A}{B}$ $\quad X = \sqrt{C}$

$Y = 2i - B$ $\quad Z = Ai$



13. Given: $(cis\theta)^4 = cos^4\theta + 4(cos^3\theta)(isin\theta) + 6(cos^2\theta)(i^2sin^2\theta) + 4(cos\theta)(i^3 sin^3\theta) + i^4 sin^4\theta$
Use the expansion above, along with DeMoivre's Theorem, to express $cos\,4\theta$ in terms of $sin\,\theta$ and $cos\,\theta$.
[2 pts] $z = cis\,\theta$

$z^4 = cis\,4\theta$

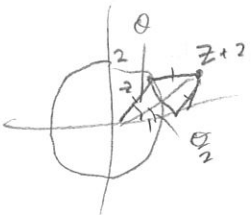
$(cis\,\theta)^4 = cis\,4\theta$

$cos4\theta = Re(cis\theta)^4$

$cos\,4\theta = Re(cos^4\theta + 4cos^3\theta(sin\theta) - 6cos^2\theta(sin^2\theta) + 4cos\theta(i^3 sin^3\theta) + sin^4\theta)$

$cos\,4\theta = cos^4\theta - 6cos^2\theta\,sin^2\theta + sin^4\theta$

14. Given: $|z| = 2$ and $Arg(z) = \theta$. Express $Arg(z(z+2))$ in terms of θ. [3 pts]



$Arg(z(z+2)) = \frac{3\theta}{2}$

15. Write a 2x2 matrix that will perform the given transformation. [1 pt each]

a) Dilation by a factor of 5

$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

b) Reflection over the y-axis

$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

c) Rotation by 180 degrees

$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

d) Map all points to the line y = 3x
(but your answer cannot be a zero matrix)

$\begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$