Analysis H 2022-2023 – Hahn / Hlasek / Tantod
AtPS Quiz 2
NO CALCULATORS

Justin Oh is growing in math. That's a fact. Orial.
Period: 7

$\frac{28}{28 \text{ pts}}$

1. Write each as a compact expression. No need to evaluate the actual number – you can just write an equivalent numerical expression. [3 pts each]

a)

$$\sum_{n=43}^{200} F_{2n}$$

$= F_{86} + F_{88} + \cdots + F_{400}$

$= \cancel{F_{87}} - F_{85} + \cancel{F_{89}} - \cancel{F_{87}} + \cdots + F_{401} - \cancel{F_{399}}$

$= \boxed{F_{401} - F_{85}}$

b)

$$\sum_{n=1}^{75}[5 + 7(n-1)]$$

$= \sum_{n=1}^{75}(7n-2)$

$= \boxed{\frac{5+(75\cdot7-2)}{2}\cdot75}$

2. Evaluate (your answer for this problem should be a single number). [3 pts]

$$\sum_{n=6}^{10} 512\left(\frac{1}{2}\right)^n = 512\cdot\left(\frac{1}{2}\right)^6 + 512\cdot\left(\frac{1}{2}\right)^7 \cdots + 512\left(\frac{1}{2}\right)^{10}$$

$\frac{S}{2} = 512\cdot\left(\frac{1}{2}\right)^7 + \cdots + 512\left(\frac{1}{2}\right)^{10} + 512\left(\frac{1}{2}\right)^{11}$

$\frac{S}{2} = 512\left(\frac{1}{2}\right)^6 - 512\left(\frac{1}{2}\right)^{11}$

$S = 512\left(\frac{1}{2}\right)^5 - 512\left(\frac{1}{2}\right)^{10} = 16 - \frac{1}{2} = \boxed{\frac{31}{2}}$

3. Simplify completely: $\binom{3n+2}{3n-1}\cdot(3!)$ Write your answer as a polynomial with integer coefficients. [3 pts]

$= \frac{(3n+2)!}{(3n-1)!\,\cancel{(3!)}}\cdot\cancel{(3!)} = (3n+2)(3n+1)(3n) = (9n^2+9n+2)(3n)$

$= \boxed{27n^3 + 27n^2 + 6n}$

4. Evaluate (give your answer as a single number): [2 pts each]

a) $\binom{-2}{500}$

$\frac{\cancel{(-2)}(-3)(-4)\cdots(+501)}{500!}$

$= \boxed{501}$

b) $\binom{6}{20} = 0$

c) $\binom{-8}{4}$

$= \frac{(-\cancel{8})(-\overset{3}{\cancel{9}})(-10)(-11)}{4\cdot\cancel{3}\cdot2\cdot1}$

$= 3\cdot10\cdot11$

$= \boxed{330}$

-0

-0 5. Prove by mathematical induction: $1 + 3 + 5 + 7 + \cdots + (2n - 1) = n^2$ for all positive integers n. [5 pts]

Assume $\sum_{x=1}^{k}(2n-1) = k^2$

these should be the same symbol. $\sum_{j=1}^{k}(2j-1)$

$\sum_{x=1}^{k}(2x-1)$

Induction step: Prove $\sum_{x=1}^{k+1}(2n-1) = (k+1)^2$

$$\sum_{x=1}^{k+1} = k^2 + (2(k+1)-1)$$

$$= k^2 + 2k + 1$$

$$= (k+1)^2$$

Base case: $1 \overset{?}{=} 1^2 = 1$ ✓

□.

6. Prove by mathematical induction: $2 \cdot 4^n + 3 \cdot 9^n$ is a multiple of 5 for all positive integers n. [5 pts]

Assume that $2 \cdot 4^k + 3 \cdot 9^k$ is a multiple of 5

Induction step: Prove $2 \cdot 4^{(k+2)} + 3 \cdot 9^{(k+2)}$ is a multiple of 5

$$2 \cdot 4^{(k+2)} + 3 \cdot 9^{(k+2)}$$

$$= 32 \cdot 4^k + 243 \cdot 9^k$$

$$= 2 \cdot 4^k + 3 \cdot 9^k + 30 \cdot 4^k + 240 \cdot 9^k$$

$$= \underbrace{2 \cdot 4^k + 3 \cdot 9^k}_{\text{multiple of 5 from assumption}} + 5\underbrace{(6 \cdot 4^k + 48 \cdot 9^k)}_{\text{multiple of 5}}$$

Base cases: $n=1$, $n=2$

$n=1$: $2 \cdot 4^1 + 3 \cdot 9^1 = 8 + 27 = 35$ is a multiple of 5

$n=2$: $2 \cdot 4^2 + 3 \cdot 9^2 = 32 + 243 = 275$ is a multiple of 5

□

F0