Analysis H – Hahn / Hlasek / Tantod
Vectors and Parametrics: Unit 4, Quiz 2
NO CALCULATOR

40
40 points

Justin Oh is crossing all t's and dotting all i's
Period: ______

For this whole quiz: Simplify all your answers completely, but you don't need to rationalize denominators.

1. Given points P(4, 2, 7), Q(3, 1, 8), and R(5, -1, 11), and plane W that contains the points P, Q, and R. [2 pts each]

a) $\overrightarrow{PQ} = \langle -1, -1, 1 \rangle$

b) $\overrightarrow{PR} = \langle 1, -3, 4 \rangle$

c) $\overrightarrow{PQ} \cdot \overrightarrow{PR} = -1 + 3 + 4$

$= \boxed{6}$

d) $proj_{\overrightarrow{PR}} \overrightarrow{PQ} = \dfrac{\overrightarrow{PR} \cdot \overrightarrow{PQ}}{|\overrightarrow{PR}|^2} \cdot \overrightarrow{PR}$

$= \dfrac{6}{26} \langle 1, -3, 4 \rangle$

$\boxed{= \frac{3}{13}\hat{i} - \frac{9}{13}\hat{j} + \frac{12}{13}\hat{k}}$

e) $\overrightarrow{PQ} \times \overrightarrow{PR} = \langle -1, -1, 1 \rangle \times \langle 1, -3, 4 \rangle$

$= \boxed{-\hat{i} + 5\hat{j} + 4\hat{k}}$

-0 f) Write a vector equation of plane W.

$W = 4\hat{i} + 2\hat{j} + 7\hat{k} + s(-\hat{i} - \hat{j} + \hat{k}) + t(\hat{i} - 3\hat{j} + 4\hat{k})$

$\langle x, y, z \rangle$ or $r =$

g) Find a unit vector that is normal to plane W.

$\hat{n} = \dfrac{PQ \times PR}{|PQ \times PR|} = \boxed{\frac{1}{\sqrt{42}}(-\hat{i} + 5\hat{j} + 4\hat{k})}$

h) Find the area of the triangle that has P, Q, and R as its vertices.

$A = \frac{1}{2}|PQ \times PR| = \boxed{\frac{\sqrt{42}}{2}}$

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2. The dot product of two (non-zero) vectors is negative. What does that mean about the angle between the vectors? [2 pt]

$\boxed{270° > \phi > 90°}$

3. The cross product of two (non-zero) vectors is 0. What does that mean about the angle between the two vectors? [2 pt]

$\boxed{\phi = 0° \text{ or } \phi = 180°}$

4. Vector $\vec{n} = \langle -2,8,5 \rangle$ is normal to plane P, and the point A(1, -3, 6) is in the plane. Give the equation of the plane in standard form (Ax + By + Cz = D). [3 pts]

$-2x + 8y + 5z = D \; ; \; (1, -3, 6)$

$-2 - 24 + 30 = D$

$D = 4$

$\boxed{-2x + 8y + 5z = 4}$

5. The vector equation of plane R is given as $\langle x, y, z \rangle = \langle 8, -5, 2 \rangle + s\langle 0,1,3 \rangle + t\langle -1,0,4 \rangle$.

a) Write the equation of the plane in standard form (Ax + By + Cz = D). [3 pts]

$\langle 0, 1, 3 \rangle \times \langle -1, 0, 4 \rangle$

$= \langle 4, -3, 1 \rangle$

$\begin{vmatrix} i & j & k \\ 0 & 1 & 3 \\ -1 & 0 & 4 \end{vmatrix} = \langle 4, -3, 1 \rangle$

$4x - 3y + z = D$

$32 + 15 + 2 = D$

$D = 49$

$\boxed{4x - 3y + z = 49}$

b) Find the distance from plane R to the point (10, 11, 12). [2 pts]

$d = \dfrac{|(4 \cdot 10) - 33 + 12 - 49|}{\sqrt{26}} = \dfrac{30}{\sqrt{26}}$

-0

6. Consider the points A(x, 11, -1), B(8, 10, -8), and C(12, 2, 1). A and B are in Plane P, and vector $\overrightarrow{AC}$ is normal to the plane. Find all possible values for x. [4 pts]

$\vec{AC} = (12 - x')\hat{i} - 9\hat{j} + 2\hat{k}$

$\boxed{x = 7 \text{ or } 13}$

$(12 - x')x - 9y + 2z = D$

$(12 - x')8 - 90 + (-16) = D$

$D = -8x' - 10$

$(12 - x')x' - 99 + (-2) = D = -8x' - 10$

$-91 = x'^2 - 20x'$

$x'^2 - 20x' + 91 = 0; \ (x' - 7)(x' - 13) = 0$

7. Given: $\vec{u} \cdot \vec{v} = 0$, $|\vec{u}| = 12$, and $|\vec{u} \times \vec{v}| = 6$. $|\vec{v}| =$ $\boxed{\frac{1}{2}}$ [1 pt]

8. $\langle 4, y, z \rangle \times \langle 9,1,3 \rangle = \langle -13,51,22 \rangle$. Solve for y and z. Show all your work to receive credit. [4 pts]

$a \times b = (a_x b_y - b_x a_y)\hat{i} + (a_z b_x - a_x b_z)\hat{j} + (a_x b_y - a_y b_x)\hat{k}$

$-13 = 3y - z$

$51 = 9z - 12$

$22 = 4 - 9y$

$\boxed{y = -2; \ z = 7}$

$z = \frac{63}{9} = 7$

$y = -2$

9. Plane P is determined by the equation $-3x + 4y + 5z = 21$. Find the rectangular equations of both planes that are parallel to Plane P and $\sqrt{2}$ units away from Plane P. [3 pts]

$(-7, 0, 0)$ is on P

$$\boxed{\begin{aligned} -3x + 4y + 5z &= 31 \\ -3x + 4y + 5z &= 11 \end{aligned}}$$

$\vec{n} = \langle -3, 4, 5 \rangle$

$\hat{n} = \frac{\langle -3, 4, 5 \rangle}{5\sqrt{2}}$

$(-7, 0, 0) + \hat{n} \cdot \sqrt{2} = \left(\frac{-38}{5}, \frac{4}{5}, 1\right)$

$(-7, 0, 0) - \hat{n} \cdot \sqrt{2} = \left(-\frac{32}{5}, -\frac{4}{5}, -1\right)$

$-3\left(-\frac{38}{5}\right) + \frac{16}{5} + 5 = D$

$-3\left(-\frac{32}{5}\right) - \frac{16}{5} - 5 = D$

$D = \frac{106 + 24}{5} + 5 = 31$

$D = \frac{80}{5} - 5 = 11$

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