Analysis H 2022/23 – Hahn / Hlasek / Tantod
Unit 9 Quiz – Calculus
No Calculators!

59
60 points

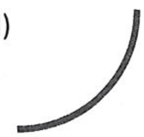
The formal definition of Justin Oh
Period: 7

Questions 1-6 are Multiple Choice. Circle the best answer. [3 pts each]

1. For a certain function $f(x)$, $\exists\delta\ such\ that\ 0 < |x - c| < \delta \rightarrow f(c) > E$. This proves that...

a) $\lim\limits_{x\to c} f(x)$ exists
b) $\lim\limits_{x\to c} f(x)$ does not exist
c) $\lim\limits_{x\to c} f(x) = E$
d) $\lim\limits_{x\to c} f(x) = 0$
e) $\lim\limits_{x\to 0} f(x) = E$

2. Carlos is doing a delta-epsilon proof for $\lim\limits_{x\to 3} f(x) = 5$, and after graphing f(x), he realizes that the maximum delta value that fulfills the definition of the limit will be to the LEFT of 3, and that in order to find the delta value, he should first solve the equation $f(x) = 5 + \varepsilon$. Which graph below best represents f(x)?

a) 
b) 
c) 
d) 

For Questions 3-7, Consider the following table, which shows y, in bloops, as a function of x, in bloops per plop.

x (bloops per plop)	10	14	15	18	22	26	40
f(x) = y (bloops)	56	60	65	70	71	74	80

3. What is the AROC of f(x) over the x-interval [10,40]?

$\frac{80-56}{30} = \frac{24}{30} = \frac{4}{5}$

a) 4/5
b) 5/4
c) 2
d) 1/2
e) 28/5

4. Which of the following is the best approximation for $f'(15)$?

?
$\frac{10}{4} = \frac{5}{2}$

a) 13/3
b) 5
c) 5/3
d) 3/5
e) 1/5

5. What are the units for $f'(x)$?

a) bloops per plop
b) plops per bloop
c) plops
d) bloops per plop^2
e) none of these

6. What are the units for the definite integral of $f(x)$?

a) bloops
b) bloops^2
c) plops
d) plops per bloop^2
e) none of these

FO

7. Fill in the blanks to complete the definition of limit. [4 pts]

$\lim\limits_{x \to c} f(x) = L$ if and only if for all values of $\underline{\ \varepsilon\ }$, there exists a $\underline{\ \delta\ }$ such that

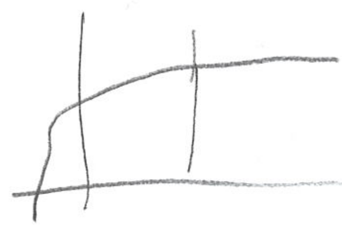
if $\underline{\ x\ }$ is within $\underline{\ \delta\ }$ of $\underline{\ c\ }$ (but not equal to $\underline{\ c\ }$), then $\underline{f(x)}$ will be within $\underline{\ \varepsilon\ }$ of $\underline{\ L\ }$.

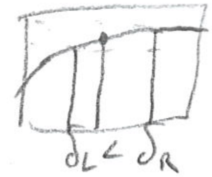
8. Consider the function $f(x) = \sqrt{x+18} - 2$

a) [2 pts] Evaluate $\lim\limits_{x \to 7} f(x) = \underline{\ 3\ }$

b) [5 pts] Write a delta-epsilon proof to prove your answer from part (a). Leave epsilon as a variable ε and include a graph in your work (that can be used to justify algebraic choices in your work). Make sure you include a conclusion statement.

$3 - \varepsilon < f(x) < \cancel{3} + \varepsilon \rightarrow \delta_L < \delta_R$





$3 - \varepsilon = \sqrt{x+18} - 2$

$5 - \varepsilon = \sqrt{x+18}$

$25 - 10\varepsilon + \varepsilon^2 = x + 18$

$x = \varepsilon^2 - 10\varepsilon + 7$

$\delta_L = |x - c|$ → since for δ_L, $x < c$

$= 7 - (\varepsilon^2 - 10\varepsilon + 7)$

$\delta_L = 10\varepsilon - \varepsilon^2$

Thus, $\forall \varepsilon > 0$, let $\delta = 10\varepsilon - \varepsilon^2$: $0 < |x - c| < \delta \rightarrow 0 < |f(x) - L| < \varepsilon$

$\therefore \lim\limits_{x \to 7} f(x) = 3$

9. Evaluate each limit, or state that it does not exist [3 pts each]

a) $\lim\limits_{x \to 3} \frac{x^2 - x - 6}{x - 3} = 5$

$\frac{(3)^2 - 3 - 6}{3 - 3} = \frac{0}{0}$

$\lim\limits_{x \to 3} = \frac{2x - 1}{1} = 5$

b) $\lim\limits_{x \to \infty} \frac{2x+3}{7x-6} = \frac{2}{7}$

c) $\lim\limits_{x \to 0} \frac{\sqrt{x^2+9} - 3}{x^2} = \frac{1}{6}$

$\frac{\sqrt{9} - 3}{0^2} = 0$

$\Rightarrow \lim\limits_{x \to 0} \frac{(x^2+9)^{1/2} - 3}{x^2} = \lim\limits_{x \to 0} \frac{\frac{1}{2} \cdot 2x \cdot (x^2+9)^{-1/2}}{2x} = \frac{1}{2\sqrt{(0^2+9)}} = \frac{1}{6}$

-0

10. [3 pts] Name the 3 criteria that comprise the Formal Definition of Continuity at a point x = c.

i) f(c) is defined

ii) $\lim_{x \to c} f(x)$ is defined

iii) $\lim_{x \to c} f(x) = f(c)$

For a function $f(x)$ to be continuous at $x=c$

11. [2 pts] Find the value(s) of *k* that will make the function continuous for all values of *x.*

$$f(x) = \begin{cases} kx^2 & if \quad x > 2 \\ kx + 4 & if \quad x \leq 2 \end{cases}$$

$k(2)^2 = k(2) + 4$

$2k = 4$

$\boxed{k = 2}$

-1 12. [3 pts] Consider the function $f(x) = 2x^2 + 4x - 9$. Does the Intermediate Value Theorem guarantee that there will be a c-value for which $f(c) = -9$ in the x-interval [-5,5]? Justify your answer with clear reasoning.

yes

1) f(x) is continuous since all polynomials are continuous

2) $-9 \in (f(-1), f(5)) = (-11, 61)$ and $-1 \in [-5, 5] \wedge 5 \in [-5, 5]$

$\therefore \exists c \in [-5, 5] : f(c) = -9$ by the IVT.

You used [-1, 5] ?

13. [2 pts each] Use the Power Rule to find the derivative of each function.

a) $f(x) = 10\sqrt{x} + \frac{3}{x^3}$

$f'(x) = 5x^{-1/2} - 9x^{-4}$

b) $g(x) = 4x^5 + 3\pi^7 - 8e^2$

$g'(x) = 20x^4$

-1

14. Consider the function $f(x) = x^2 - 5x - 7$

a) [3 pts] Find the AROC over the x-interval [6,8]. Show all your work, including the difference quotient, for full credit.

$$AROC\Big|_6^8 = \frac{f(8)-f(6)}{8-6}$$

$$= \frac{17+1}{2} = \boxed{9}$$

b) [4 pts] Use the Formal Definition of the Derivative at a Point (FDoDaP) to find $f'(6)$. For full credit, you must show the initial set up of FDoDaP and accurate steps to show the evaluation. If you get the correct answer using the Power Rule with no further justification, you will lose 2 points.

$$f'(6) = \lim_{x \to 6} \frac{f(x) + f(6)}{x-6}$$

$$= \lim_{x \to 6} \frac{x^2-5x-7+1}{x-6}$$

$$= \lim_{x \to 6} \frac{\cancel{(x-6)}(x+1)}{\cancel{x-6}}$$

$$= 6+1$$

$$f'(6) = \boxed{7}$$

c) [3 pts] Find the equation of the line (in point-slope form) that has a positive slope, is tangent to $f(x)$, and also contains the point (1, -20).

$$f'(x) = \frac{f(x)+20}{x-1}$$

$$2x-5 = \frac{x^2-5x+13}{x-1}$$

$$(2x-5)(x-1) = x^2-5x+13$$

$$2x^2-7x+5 = x^2-5x+13$$

$$x^2-2x-8=0$$

$$x = 4, -2 \quad f'(-2) \ngtr 0$$

$$f'(4) = 3$$

$$f'(-2) = -9$$

$$\boxed{y+20 = 3(x-1)}$$

-0