Analysis Honors – Hlasek / Hahn / Tantod
Unit 3: Polar and 3D Graphing
Big Quiz – No Calculators

65

65 points

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Period: 7

Part I: Polar Graphing

Questions 1-3 are Multiple Choice: Circle the best answer for each problem. [2 pts each]

1. The graph of $r = -6\sin 12\theta$ is a rose curve with ____________

   a) 24 petals, one of them on the x-axis
   b) 24 petals, one of them on the y-axis
   c) 24 petals, with no petal on an axis
   d) 12 petals, one of them on the y-axis
   e) 12 petals, one of them on the x-axis
   f) 12 petals, with no petal on an axis

2. The graph of $r = \frac{12}{4+5\cos\theta}$ is a ______________

   $4r + 5r\cos\theta = 12$

   $4\sqrt{x^2+y^2} + 5x = 12$

   $4\sqrt{x^2+y^2} = 12 - 5x$

   $16x^2+y^2 = (-5x+12)^2$

   $y^2 = 9x^2 - 120x + 144$

   a) limacon
   b) parabola
   c) ellipse
   d) hyperbola
   e) rose curve
   f) none of the above

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3. Consider the system of equations: $r = 8\sin 2\theta$ and $r^2 = 49\cos 2\theta$. How many points of intersection are there?

   a) 4
   b) 5
   c) 8
   d) 9
   e) more than 9
   f) less than 4

4. Consider the polar points A $\left(6, \frac{7\pi}{6}\right)$ and B $\left(-4, \frac{2\pi}{3}\right)$.

   a) Graph and label the points on the polar plane on the right. [2]

   b) Find the length of line segment AB. Show all your work, and give your answer in exact simplified form. [3]

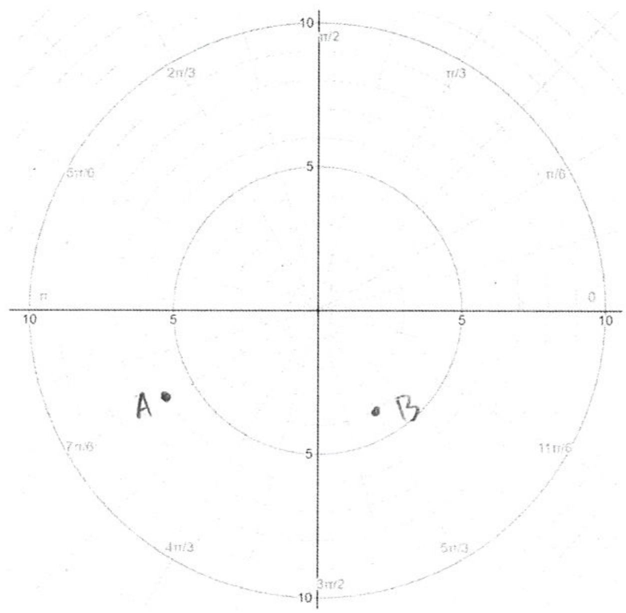
   A → rect. $(-3\sqrt{3}, -3)$, B → rect. $(2, -2\sqrt{3})$

   $AB = \sqrt{(2+3\sqrt{3})^2 + (3-2\sqrt{3})^2}$

   $= \sqrt{4+27+\cancel{12\sqrt{3}}+9+12-\cancel{12\sqrt{3}}}$

   $= \sqrt{4+27+9+12}$

   $= \sqrt{52}$



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5. Consider the equation $r = 4 + k\sin\theta$. Give ALL values of k for which the graph will be a limacon with an inner loop. [4 pts]

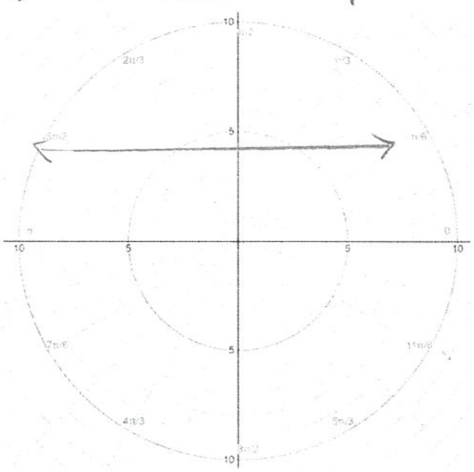
$\boxed{k > 4 \text{ or } k < -4}$

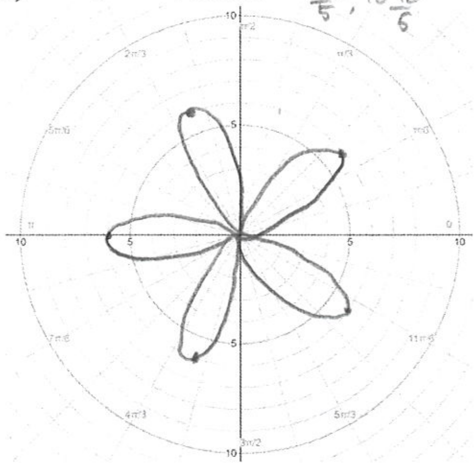
6. Graph each function. Then classify it according to its most specific name. [3 pts per graph, 1 pt for name]

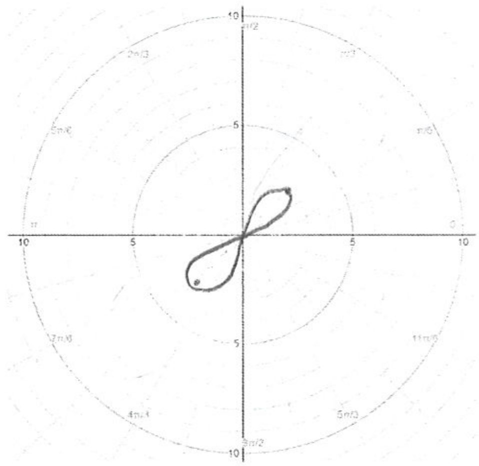
a) $r = 4\csc\theta$ $\quad r = \frac{4}{\sin\theta}$ $\quad r\sin\theta = 4$ $\quad y = 4$

b) $r = -6\cos 5\theta$ $\quad \theta, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, \frac{10\pi}{6}$

c) $r^2 = 9\sin 2\theta$





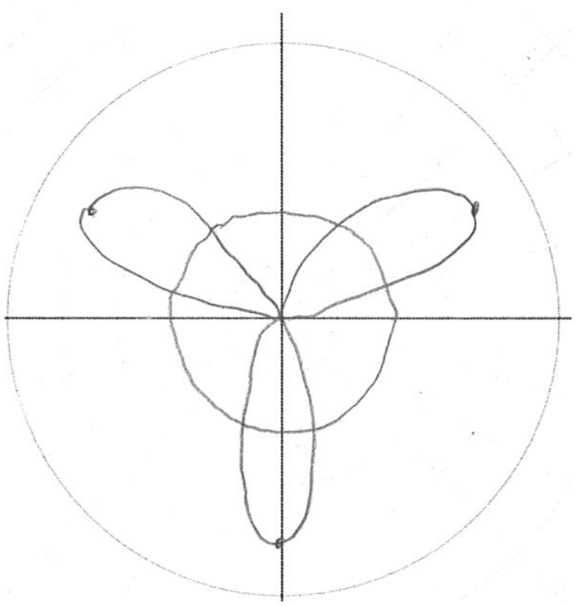


Name: line

Name: cosine rose curve w/ 5 petals

Name: lemniscate

7. Find all the points of intersection for the system of equations: $r = 4\sin(3\theta)$ and $r = 2$ (the graphing space on the left is for your work, but will not be graded) [4 pts]



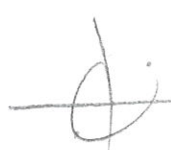
$4\sin 3\theta = 2$

$\sin 3\theta = \frac{1}{2}$

$\theta = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

$4\sin(3\theta + 3\pi) = -2$

$-4\sin(3\theta) = -2$



$\boxed{(2, \frac{\pi}{18}); (2, \frac{5\pi}{18}); (2, \frac{13\pi}{18}); (2, \frac{17\pi}{18}), (2, \frac{25\pi}{18}); (2, \frac{29\pi}{18})}$

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Part II: 3D graphing

8. For each 3D equation below, write the letter that represents the best name for the graph. It's possible to use the same answer choice more than once. [2 pts each]

A: Hyperbolic Cylinder B: Hyperboloid of 1 Sheet C: Hyperboloid of 2 Sheets D: Ellipsoid

E: Elliptic Cone F: Hyperbolic Paraboloid G: Elliptic Paraboloid H: None of the Above

$x^2 - (y^2 - y) = z^2$

$x^2 = y^2 - y + z^2$

$x^2 = (y - \frac{1}{2})^2 + z^2 - \frac{1}{4}$

△ i) $x^2 - y^2 + y = z^2$ __B__ iv) $x + y^2 - z^2 = 0$ __F__

$x^2 - (y^2 - y) - z^2 = 0$

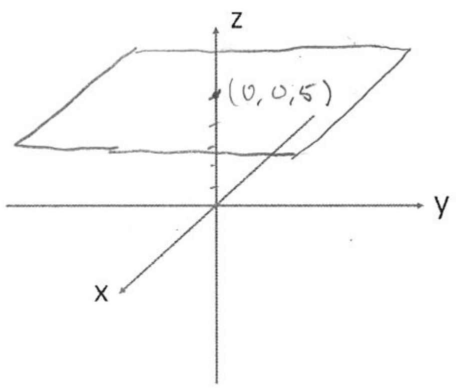
ii) $z^2 + y^2 = x + 3$ __G__ v) $x^2 + y^2 + z^2 = -8$ __H__

iii) $x^2 - z^2 = 7$ __A__ vi) $9y^2 + z^2 = x^2$ __E__

9. Sketch each 3D shape. In your drawing, label at least one **point** in each graph that lies in the figure, and then state the name of the shape. [3 pts each graph, 1 pt for name]

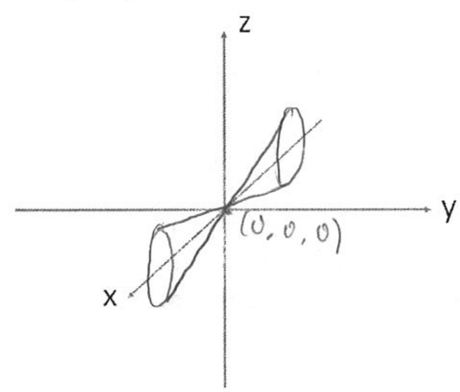
$y^2 + \frac{z^2}{4} = \frac{x^2}{4}$

a) $z = 5$



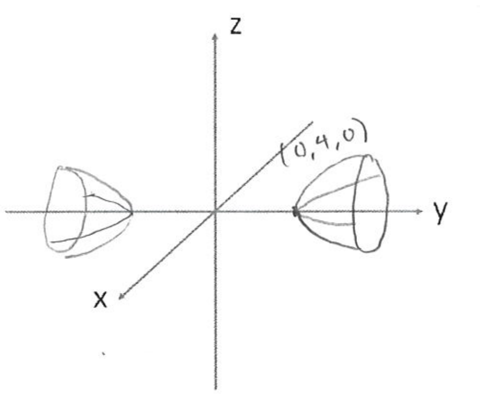
Name: plane

b) $\frac{x^2}{4} - \frac{z^2}{9} = y^2$ $\frac{x^2}{4} - y^2 - \frac{z^2}{9} = 0$



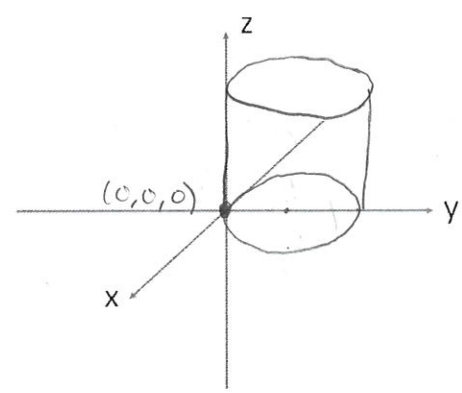
Name: eliptic cone

c) $-\frac{x^2}{9} + \frac{y^2}{16} - \frac{z^2}{9} = 1$ $\frac{x^2}{9} + \frac{z^2}{9} - \frac{y^2}{16} = -1$



Name: hyperboloid of 2 sheets

d) $\frac{x^2}{9} + \frac{(y-5)^2}{25} = 1$



Name: elliptic cylinder

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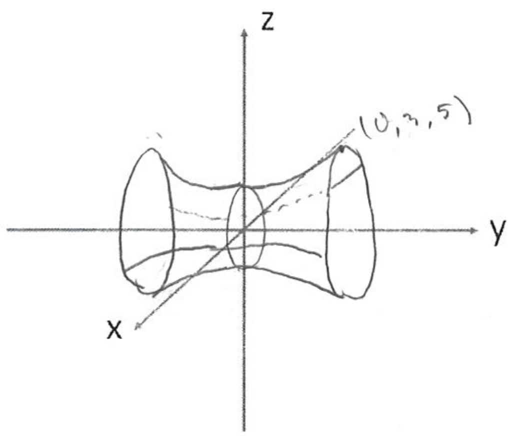
For Questions 10 and 11:
The equations of the 3D graphs can be written in the form: $Ax^2 + Bx + Cy^2 + Dy + Ez^2 + Fz + G = 0$.
Convert your equations into this form, and solve for the coefficients A, B, C, D, E, F, and G.

10. Sketch a picture of a hyperboloid of one sheet whose **xz trace** is a circle with radius 4, centered on the origin. Then solve for the coefficients of the equation if the graph also hits the point (0, 3, 5). Show all your work to receive full credit. [2 pts for sketch, 3 pts for work and answer]

Sketch:



Work shown here:

$\frac{x^2}{16}+\frac{z^2}{16}-\frac{y^2}{16}=1 \longleftarrow y=0,\ x^2+z^2=16$

$x^2+z^2-y^2=16$

$x^2-y^2+z^2-16=0$

(0, 3, 5)

$0^2+C(3^2)+25=16$

$C=\frac{16-25}{9}$

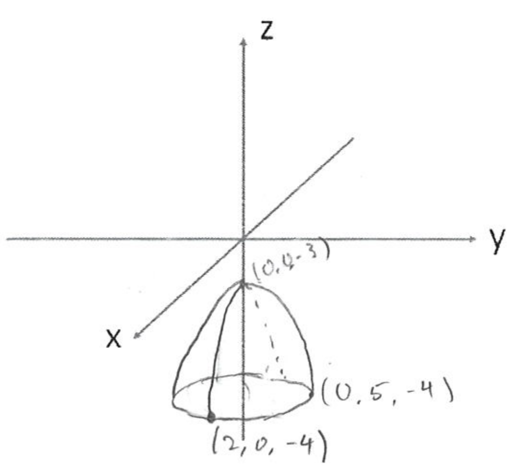
$=-1$

Coefficients of the equation:

A = 1 B = 0 C = -1 D = 0 E = 1 F = 0 G = -16

11. Sketch a picture of an elliptical paraboloid that opens along the negative z-axis, and has a vertex at (0, 0, -3). Then solve for the coefficients of the equation if the graph also hits the points (2, 0, -4) and (0, 5, -4). Show all your work to receive full credit. [2 pts for sketch, 3 pts for work and answer]

Sketch:



Work shown here:

$-z-3=\frac{x^2}{4}+\frac{y^2}{25}$

$\frac{x^2}{4}+\frac{y^2}{25}+z+3=0$

(2, 0, -4)

$4-3=Ax^2=A\cdot 4$

$A=\frac{1}{4}$

(0, 5, -4)

$1=C(5^2)$

$C=\frac{1}{25}$

Coefficients of the equation:

A = $\frac{1}{4}$ B = 0 C = $\frac{1}{25}$ D = 0 E = 0 F = 1 G = 3

-0