Analysis H – Hahn, Hlasek, Tantod			Hope I don't get a mid grade:
Midterm Exam 1 2023-2024		→	Period:
NO CALCULATORS	60 pts	100 pts	

## Part 1: Algebra Through Problem Solving

## Problems 1 – 5 are multiple choice. Circle the best answer for each question. [3 pts each]

1. The expansion of  $\left(\frac{1}{2}x - 2y\right)^7$  has 8 terms (in the 1<sup>st</sup> term, there is an  $x^7$ ). What is the coefficient of the 5<sup>th</sup> term?

a) 70 b) 17.5 c) 35 d) -70 e) -168

2. If a statement can be shown to be true for n = 1 and then, through mathematical induction, proved true for any case n = k + 3, then it can be concluded that the statement is true for...

- a) All integer values of *n*
- b) All integer values of  $n \ge 1$
- c) All values of *n*, where *n* is a positive odd integer
- d) All values of *n*, where  $\left(\frac{n+2}{3}\right)$  is a positive integer
- e) All values of *n* where *n* is a triangular number

3. In Pascal's Triangle, the 1<sup>st</sup> diagonal is all 1's and the 2<sup>nd</sup> diagonal shows the Natural Numbers. Which of the following expressions is the best representation of the numbers in the 3<sup>rd</sup> diagonal?

a)  $a_n + a_{n-1}$  b)  $F_n$  c)  $\frac{n(n+1)}{2}$  d)  $n^2$  e) 2n - 1

4. Given: *x* and *y* are distinct (different) positive integers. If *a* is the arithmetic mean of *x* and *y*, and *g* is the geometric mean of *x* and *y*, then which of the following is true?

a) a > g b)  $a \ge g$  c) a < g d)  $a \le g$  e) none of the above

5. Evaluate:  $\sum_{n=3}^{33} 128 \left(\frac{1}{2}\right)^n =$ 

a) 
$$32\left(1-\left(\frac{1}{2}\right)^{31}\right)$$
 b)  $\frac{64\left(1-\left(\frac{1}{2}\right)^{30}\right)}{1-\frac{1}{2}}$  c)  $\frac{16\left(1-\left(\frac{1}{2}\right)^{30}\right)}{1-\frac{1}{2}}$  d)  $128\left(\frac{1}{2}\right)^{33}$  e)  $16\left(\frac{1}{2}\right)^{33}$ 

## Questions 6-10 are Free Response. Justify all your answers with work and/or explanations.

6. Given:  $F_n$  is the n<sup>th</sup> Fibonacci number. Write  $F_7 + F_9 + F_{11} + F_{13} + \dots + F_{77} + F_{79}$  as a compact expression (without the "…", and without using Sigma). [3 pts]

7. Refer to the odd-number triangle on the right, where the first row is [1], and the second row is [3, 5]. The n<sup>th</sup> row of the odd-number triangle has n terms, and the middle term of the row is equal to  $n^2$ . Find an expression for the 4<sup>th</sup> term of the n<sup>th</sup> row. [3 pts]

1	
3 5	
7 9 11	
13 15 17 19	

8. Simplify completely:  $\binom{x+y}{2} - \binom{x}{2} - \binom{y}{2}$  [3 pts]

9. Write each expression as a single exponent, choose number, or factorial. [1 pt each]

a) 
$$\binom{15}{0} + \binom{16}{1} + \binom{17}{2} + \binom{18}{3} + \dots + \binom{82}{67} =$$

b) 
$$\binom{32}{1} + \binom{32}{3} + \binom{32}{5} + \binom{32}{7} + \dots + \binom{32}{15} =$$

c) 
$$\binom{45}{7} + 2\binom{45}{8} + \binom{45}{9} =$$

10. Consider the sequence of numbers: <u>1, 2, 3</u>, <u>2, 3, 4</u>, <u>3, 4, 5</u>, <u>4, 5, 6</u>, <u>5, 6, 7</u>, <u>6, 7, 8</u>, <u>7, 8, 9</u>, <u>8, 9, 10</u>, ... (The underlines are there just to help you see the pattern). If  $a_1 = 1$ ,  $a_2 = 2$ , and  $a_{12} = 6$  then find...

- a) [1 pt]  $a_{25} =$
- b) [2 pts]  $a_{337}$  = (show your work for how you found this one)

## Part 2: Probability

- Problems 1 5 are multiple choice. Circle the best answer for each question. [2 pts each]
  - 1. Refer to the diagram on the right, which shows the results of a survey question "which of the following activities do you enjoy?"

 $P((SUT) \mid J) =$ 

- a) 23/27 b) 32/27 c) 47/51
  - d) 27/51 e) 9/27



- 2. Two standard 6-sided dice are rolled. What is the probability that the sum of the dice is less than or equal to 5, given at least one of the dice is a 3? [2pts]
  - a) 2/5 b) 4/11 c) 8/11 d) 2/11 e) 2/9
- 3. Which of the sets below corresponds to the shaded area of the Venn Diagram below?



## Multiple Choice (continued). Circle the best answer for each question. [2 pts each]

- 4. Suppose your team has a 40% chance of winning or tying today's game and has a 30% chance of winning today's game. Assuming that the three possible outcomes are "Win", "Loss", and "Tie", what is the probability that today's game will be a tie?
  - a) 30% b) 70% c) 40% d) 10% e) 60%
- 5. Two standard, 6-sided dice are rolled. One die is red and the other is green. Events RE and GO are defined as follows:

RE = the Red die shows an Even number GO = the Green die shows an Odd number

By these definitions, events RE and GO are...

- a) Mutually exclusive and independent c) Independent but not mutually exclusive
- b) Mutually exclusive, but not independent
- d) Not Independent and Not mutually exclusive
- e) Only independent if the red die is 18 years old

# Questions 6-10 are Free Response. For these problems, no need to justify your answers unless specified in the instructions.

- 6. A trick coin is designed so the probability it will land "heads" is 2/3.
  - a) If the coin is flipped 3 times, the probability the first 2 flips are heads and the third flip is tails is: [1 pts]
  - b) I mix the trick coin with 2 other normal "fair" coins and shuffle them around. I pick a random coin out of the three and start flipping it. What it the probability that I picked a "fair" coin given that I flip a tail on the first flip? Show the work that leads to your answer. [2 pts]

- 7. Suppose we are dealt five cards from an ordinary 52-card deck. What is the probability that
  - a) We get all four aces, plus the king of spades? [2 pts]
  - b) All five cards are spades? [2 pts]
  - c) We get no pairs (i.e. all five cards are different values)? [2 pts]
- 8. 30% of kids like to play soccer and 60% of kids like to ride their bike. If the events "like to play soccer" and "like to ride a bike" are independent of each other,
  - a) What percent of students like both soccer and riding a bike? [1 pt]
  - b) Make a Venn diagram to help you model the problem [0 pts]

c) What percentage of kids like to ride a bike, but not play soccer?[1 pt]

d) What is the probability that they like soccer, given that they like to ride a bike? [1 pt]

9. I write the 8 letter in "ANALYSIS" in random order. What is the probability that the random word will have the letters SIS next to each other, in any order? [2 pts]

- 10. While walking in the forest, you encounter a leprechaun who has two bags filled with coins. Bag A has 3 silver coins and 7 gold coins, and Bag B has 6 silver coins and 2 gold coins. You cannot tell the bags apart from the outside. The leprechaun offers to let you randomly select one of the bags and then randomly select one coin from that bag.
  - a) Make and fully label a tree diagram to help you model the problem [0 pts]

- b) Find the probability that the coin you choose is gold. [2 pts]
- c) Find the probability that the coin you chose came from Bag B, given that it is gold. [2 pts]
- d) If the gold coins are worth \$20 while the silver coins are only worth \$5, what is the expected value of the coin that you draw? [2 pts]