Analysis Honors - Hahn / Hlasek / Tantod
Midterm Exam 2, 2023-2024

Completely normal student: _____
Period: _____

_____
100 points

Section 1: Polar Graphing [25 points]

Questions 1-5 are Multiple Choice. Circle the best answer. [3 points each]

1. The graph of $r = 4\sin 3\theta$ contains the polar point $\left(k, \frac{\pi}{8}\right)$. Which of the following points is also on the graph?

a) $\left(k, \frac{3\pi}{8}\right)$ b) $\left(-k, \frac{\pi}{8}\right)$ c) $\left(k, \frac{7\pi}{8}\right)$ d) $\left(k, -\frac{\pi}{8}\right)$ e) $\left(k, \frac{5\pi}{8}\right)$

2. Which equation is a convex limacon?

a) $r = 8 + 3\cos\theta$ b) $r = 3 + 8\cos\theta$ c) $r = 8 + 12\cos\theta$

d) $r = 8 + 7\cos\theta$ e) $r = -8\cos\theta$

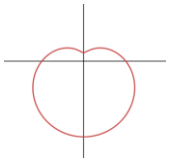
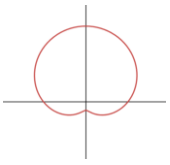
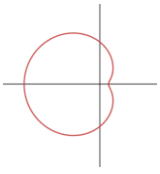
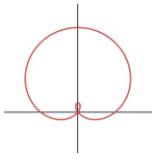
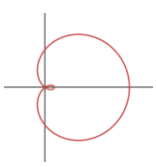
3. The graph of $r = \frac{8}{2+\sin\theta}$ is a(n):

a) line b) plane c) parabola d) hyperbola e) ellipse

4. Consider the system of equations $r = 5\cos 4\theta$ and $r = 4$. How many solutions to the system are there?

a) 5 b) 4 c) 8 d) 16 e) 10

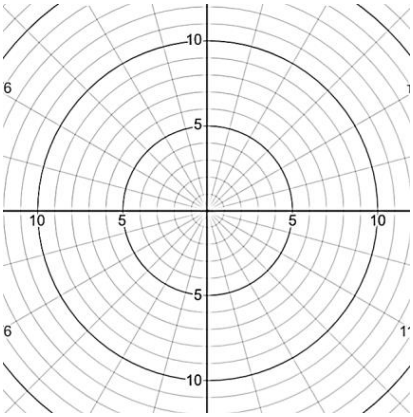
5. Which best represents the graph of $r = -5 + 4\sin\theta$?

a)  b)  c)  d)  e) 

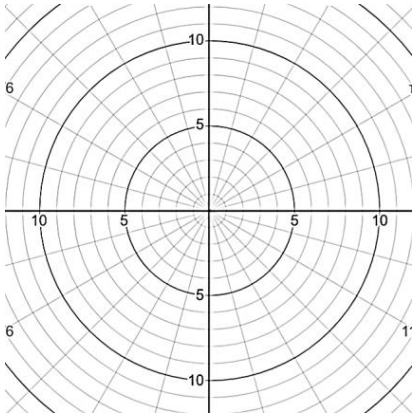
Free Response Section:

6. Graph each equation. For full credit, your intercepts must be accurate. [2 pts each]

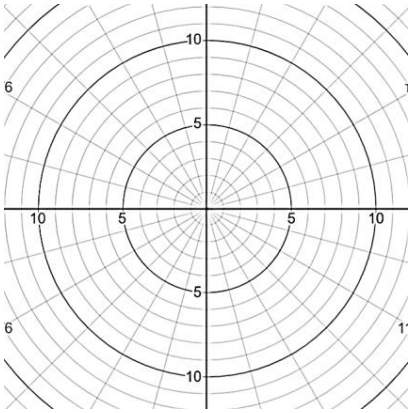
a) $r = 4 - 4\sin\theta$



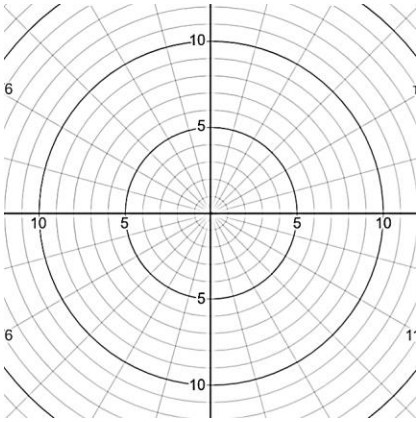
b) $r = -8\sin 3\theta$



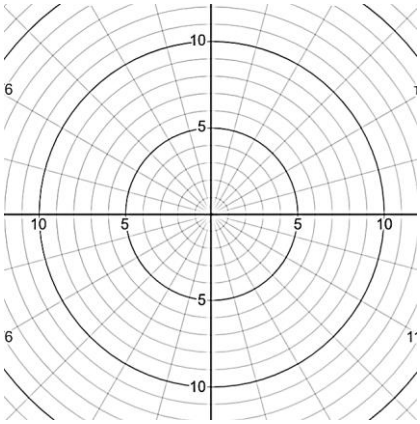
c) $r^2 = 9\cos 2\theta$



e) $r = 9\cos\theta$



f) $r = 9\cos 2\theta$



Section 2: 3D Graphing [25 points]

7. Write the equation of a parabolic cylinder that is parallel to the y-axis (extends in the y-direction) and contains the points (1, 2, 3) and (4, 5, 6). Your equation may not contain fractions or decimals (only integer coefficients). [3 pts]

Word Bank:

A = Line **B = Plane** **C = Parabolic Cylinder** **D = Hyperbolic Cylinder**

E = Hyperboloid of One Sheet **F = Hyperboloid of Two Sheets** **G = Hyperbolic Paraboloid**

H = Elliptic Cylinder **I = Elliptic Paraboloid** **J = Elliptic Cone** **K = Ellipsoid**

For questions 8-14, write the LETTERS from the word bank above to answer the question. Letters may be used more than once on this page, and some questions have multiple answers.

8. The graph of $x^2 + y^2 - z^2 = 8$ is a _____. [3 pts]

9. The graph of $y + x^2 = z^2$ is a _____. [3 pts]

10. The graph of $5x^2 + z^2 = -y^2 + 10$ is a _____. [3 pts]

11. One cross section of this shape can be a parabola, and another cross section of this shape can be an ellipse (Two answers, and you must give both for full credit. You will lose points for any wrong answer). [2 pts]

_____ and _____

12. One cross section of this shape can be an ellipse, and another cross section of this shape can be two parallel lines (two answers, and you must give both for full credit. You will lose points for any wrong answer). [2 pts]

_____ and _____

13. The xy trace of these shapes can be a hyperbola (multiple answers, and you must give ALL of them for full credit. You will lose points for any wrong answer). [2 pts]

_____

14. The yz trace of these shapes can be a single line (multiple answers, and you must give ALL of them for full credit. You will lose points for any wrong answer). [2 pts]

_____

15. Write the equation of a sphere with center (2, 3, 4) that contains the origin (0, 0, 0). [2 pts]

16. Sketch a hyperboloid of 1 sheet whose xz-trace is an ellipse centered on the origin. Then write an equation to match your sketch.

Sketch: [2 pt] Equation: [1 pt]

Section 3: Vectors and Parametric Equations [50 points]

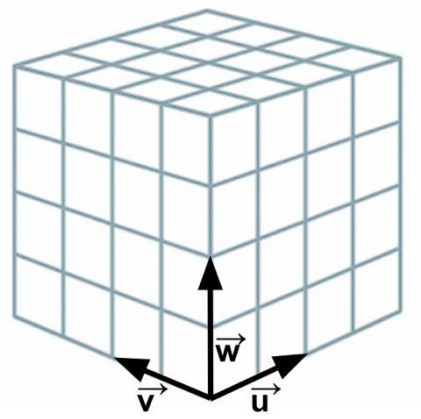
Questions 17-18 are Multiple Choice. Circle the best answer [3 pts each]

17. The diagram on the right shows a $4 \times 4 \times 4$ cube built from unit cubes and vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$.

a) $\vec{w} = 2(\vec{v} \times \vec{u})$ b) $2\vec{w} = \vec{u} \times \vec{v}$

c) $\vec{w} = 2(\vec{u} \times \vec{v})$ d) $2\vec{w} = \vec{v} \times \vec{u}$

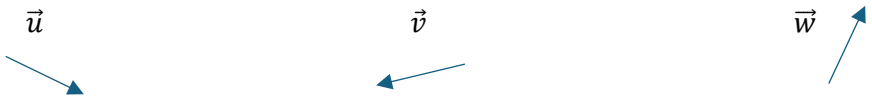
e) None of the above statements are true.



18. In the $4 \times 4 \times 4$ cube on the right, $\vec{w} \cdot (\vec{v} \times \vec{u}) =$

a) 8 b) -8 c) 16 d) -16 e) 4

For Questions 19-20, use the vectors given below, and $\vec{u}$, $\vec{v}$, and $\vec{w}$ are all unit vectors. Questions 19-20 are Multiple Choice. Circle the best answer [3 pts each]



19. Given $\vec{a} = \vec{u} - 2\vec{v} + \vec{w}$, the best representation for $\frac{\vec{a}}{|\vec{a}|}$ is

a)  b)  c)  d)  e) 

20. Assuming this test paper is flat on your desk, the best description for $\vec{w} \times proj_{\vec{v}}\,\vec{u}$ is

a) A vector pointing up towards the ceiling with magnitude greater than 1

b) A vector pointing up towards the ceiling with magnitude less than 1

c) A vector pointing down towards the floor with magnitude greater than 1

d) A vector pointing down towards the floor with magnitude less than 1

e) A null vector (magnitude = 0)

For the rest of this test, simplify all numerical answers as much as possible but leave them in exact form (with radicals where applicable). Simplify all fractions and rationalize all denominators where applicable.

21. Find the values of p and q. Give each answer as a single number. Simplify as much as possible.

a) $\begin{vmatrix} -4 & 2 \\ p & 7 \end{vmatrix} = -12$ [2 pt]

b) $\begin{vmatrix} 1 & -2 & 3 \\ 2 & 0 & 3 \\ 1 & q & 4 \end{vmatrix} = 25$ [3 pts]

22. Given points $K(-1, 0, 2)$, $L(2, -4, 3)$, $M(-4, -2, 3)$, find…

a) $\overrightarrow{KL} =$ _____ [1pt]

b) $\overrightarrow{KM} =$ _____ [1pt]

c) $\overrightarrow{KL} \times \overrightarrow{KM} =$ _____ [2pts]

d) $\overrightarrow{KM} \times \overrightarrow{KL} =$ _____ [2pts]

e) Write an equation of the plane KLM in the form $ax + by + cz + d = 0$, where a, b, c, and d are integers. [3 pts]

f) Find the coordinates of a point, where line with equation $\langle x, y, z\rangle = \langle 7, \frac{1}{2}, 1\rangle + t\langle 10, -9, 2\rangle$ intersects the plane KLM. [3 pts]

23. Let P be a plane with equation $x + 2y - z = -4$. Given that the distance of point $F(-3, k, -1)$ from plane P is $\sqrt{6}$, find all possible values of k. [3pts]

24. Given points $A(1, 2, -3)$, $B(2, 0, -2)$, $C(1, 4, -1)$, find…

a) $\overrightarrow{AB} =$ _____ [1pt]

b) $\overrightarrow{AC} =$ _____ [1pt]

c) the area of the triangle ABC [4pt]

d) $comp_{\overrightarrow{AB}} \overrightarrow{AC}$ [3 pts]

e) the distance from point C to the line AB [3 pts]

25. Consider planes R and S, given below. [3 pts]

Plane R: $4x + 2y - z + 19 = 0$

Plane S: $x = 1 + 6s - t \;\; ; \;\; y = 3 - 2s + 2t \;\; ; \;\; z = 14 - s$

Are planes R and S "parallel", "perpendicular", or "neither"? _____

Justify your answer.

26. Given the 2-D parametric equation, sketch the graph over the given t interval. [3 pts]

$x = -5\,sin\,t$
$y = 3\,cos\,t + 2$
$t: \left(0, \frac{3\pi}{2}\right)$