Analysis H – Hahn / Hlasek / Tantod
Unit 8: Limits, Quiz 1 on Sequences
No Calculators

Name______________
Period: _____

____
26 points

1. Answer True or False for each statement. [1 pt each]

   a) If a sequence is always decreasing and bounded below, it must converge. ____________________

   b) If a sequence has an upper and a lower bound, then it the sequence must have a limit. _______________

   c) If a sequence is always increasing and does NOT have an upper bound, it must be divergent. ___________

   d) If a sequence $\{a_n\}$ is everywhere decreasing, then $a_n \geq a_{n+1}$ for all values of n. ____________________

   e) If $a_n \leq b_n \leq c_n$ for all n, and a_n converges to -1 and c_n converge to 1, then b_n converges to 0. ___________

   f) If $a_n > -n$ for all n, this proves that $\{a_n\}$ converges. ______________

2. a) Is the sequence $\left\{\frac{2^n}{n!}\right\}$ always decreasing? If so, prove it algebraically. If not determine (and justify) the interval over which it is decreasing. [3 pts]

   b) Do one additional thing to prove that the sequence above converges. Include a conclusion statement. [3 pts]

3. Find the limit of each sequence, or say "diverges" if the sequence diverges. No formal proof is required. [2 pts each]

   a) $a_n = 2n - \frac{7}{n}$

   b) $b_n = \frac{b_{n-1}}{2} - 1;\quad b_1 = 3$

   c) $c_n = \frac{n}{3^n}$

4. a) For the sequence $\left\{\frac{2n+1}{5n+4}\right\}$, state the value to which the sequence converges, or state that the sequence diverges. CLEARLY justify your answer with a neighborhood proof for general ε. If your work is correct but is difficult to interpret, you may not receive full credit. [4 pts]

the **value** to which the sequence converges (or say "diverges"): ___________

Show your work here:

b) For the sequence $\left\{\frac{\cos(4n)}{5^n}\right\}$, state the value to which the sequence converges, or state that the sequence diverges. CLEARLY justify your answer with a proof. For your proof, you MUST use one of the following tests: the squeeze theorem OR the big theorem (bounded above/below and always increasing/decreasing theorem) OR the comparison principle. If your work is correct but is difficult to interpret, you may not receive full credit. [4 pts]

Test/Principle used: ______________________

the **value** to which the sequence converges (or say "diverges"): ___________

Show your work here: