Analysis H – Hahn / Hlasek / Tantod
Unit 8: Limits, Quiz 1 on Sequences
No Calculators

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26

26 points

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Period: 6

1. Answer True or False for each statement. [1 pt each]

   a) If a sequence is always decreasing and bounded below, it must converge. T ✓

   b) If a sequence has an upper and a lower bound, then it the sequence must have a limit. F ✓

   c) If a sequence is always increasing and does NOT have an upper bound, it must be divergent. T ✓

   d) If a sequence $\{a_n\}$ is everywhere decreasing, then $a_n \geq a_{n+1}$ for all values of n. T ✓

   e) If $a_n \leq b_n \leq c_n$ for all n, and a_n converges to -1 and c_n converge to 1, then b_n converges to 0. F ✓

   f) If $a_n > -n$ for all n, this proves that $\{a_n\}$ converges. F ✓

2. a) Is the sequence $\left\{\frac{2^n}{n!}\right\}$ always decreasing? If so, prove it algebraically. If not determine (and justify) the interval over which it is decreasing. [3 pts]

$\frac{2^n}{n!} \geq \frac{2^{n+1}}{(n+1)n!}$ $\quad 1 \geq \frac{2}{n+1}$ $\quad n+1 \geq 2$ $\quad \boxed{n \geq 1}$

For all values $n \geq 1$, the sequence is decreasing ✓

b) Do one additional thing to prove that the sequence above converges. Include a conclusion statement. [3 pts]

$\frac{2^n}{n!} \geq 0$

$2^n \geq 0$, because n is positive integers, all values of n make this inequality true, meaning it has a lower bound at 0 so it converges. (Big Theorem) ✓

3. Find the limit of each sequence, or say "diverges" if the sequence diverges. No formal proof is required. [2 pts each]

   a) $a_n = 2n - \frac{7}{n}$

   $\infty - 0 = \infty$

   $\boxed{\text{Diverges}}$ ✓

   b) $b_n = \frac{b_{n-1}}{2} - 1$; $b_1 = 3$

   $\frac{3}{1}, \frac{1}{2}, \frac{-3}{4}, \frac{-11}{8}, \frac{-27}{16}$

   $\times 2 \quad \times 2 \quad \times 2 \quad \times 2$

   $3, 1, -3, -11, -27 \leftarrow 5 - 2^n$

   $1, 2, 4, 8, 16 \leftarrow 2^{n-1}$

   $\frac{5-2^n}{2^{n-1}} \cdot \frac{2}{2} = \frac{2(5-2^n)}{2^n} \begin{matrix} \leftarrow -\infty \\ \leftarrow \infty \end{matrix}$ $\quad -1 \cdot 2 = -2$

   $\boxed{\text{Converges to } -2}$ ✓

   c) $c_n = \frac{n}{3^n}$ ← increases much faster, really small fraction

   $\boxed{\text{Converges to } 0}$ ✓

   6/6

4. a) For the sequence $\left\{\frac{2n+1}{5n+4}\right\}$, state the value to which the sequence converges, or state that the sequence diverges. CLEARLY justify your answer with a neighborhood proof for general ε. If your work is correct but is difficult to interpret, you may not receive full credit. [4 pts]

the **value** to which the sequence converges (or say "diverges"): $\frac{2}{5}$

Show your work here:

$$\frac{2}{5}-\epsilon < \frac{2n+1}{5n+4} < \frac{2}{5}+\epsilon$$

$$2n - 5n\epsilon + \frac{8}{5} - 4\epsilon < 2n+1 \qquad 2n + 5n\epsilon + \frac{8}{5} + 4\epsilon > 2n+1$$

$$5n\epsilon > \frac{3}{5} - 4\epsilon \qquad 5n\epsilon > -4\epsilon - \frac{3}{5}$$

$$n > \frac{\frac{3}{5}-4\epsilon}{5\epsilon} \qquad n > \frac{-4\epsilon - \frac{3}{5}}{5\epsilon}$$

← larger value so covers the other

For $n \geq \left\lceil \frac{\frac{3}{5}-4\epsilon}{5\epsilon} \right\rceil$, the sequence will be within ϵ of $\frac{2}{5}$ meaning it in the neighborhood therefore converges to $\frac{2}{5}$.

✓

b) For the sequence $\left\{\frac{\cos(4n)}{5^n}\right\}$, state the value to which the sequence converges, or state that the sequence diverges. CLEARLY justify your answer with a proof. For your proof, you MUST use one of the following tests: the squeeze theorem OR the big theorem (bounded above/below and always increasing/decreasing theorem) OR the comparison principle. If your work is correct but is difficult to interpret, you may not receive full credit. [4 pts]

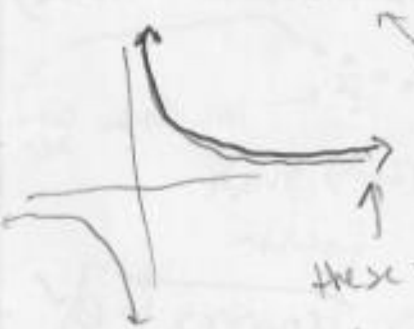
Test/Principle used: squeeze

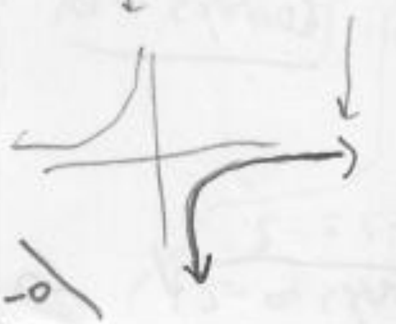
the **value** to which the sequence converges (or say "diverges"): 0

Show your work here:

$$-\frac{1}{n} \leq \frac{\cos(4n)}{5^n} \leq \frac{1}{n}$$

1. $\frac{1}{n}$ and $-\frac{1}{n}$ both converge to 0 (denominator goes to ∞)
2. $\cos(4n)$ can only have values between -1 and 1 inclusive
3. 5^n will always be greater than n because n is positive integers.
4. Because numerator ($\cos(4n)$) will either be closer to zero or the same as -1 and 1 and the denominator (5^n) will always be larger than n, $\frac{\cos(4n)}{5^n}$ will have infinite values in between $\frac{1}{n}$ and $-\frac{1}{n}$.
5. This means $\frac{\cos(4n)}{5^n}$ will converge to 0 like $\frac{1}{n}$, $-\frac{1}{n}$ do.





see 1

✓

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