

No calculators. Leave answers in factorial, exponent, or "choose" form.

1. The 8 letters of the word REMEMBER are arranged in a line.

a) Find the number of different arrangements if there are no restrictions. As a hint: one possible arrangement is BEEEMMRR [2 pts]

$$\frac{8!}{3!2!2!} \checkmark$$

b) Find the number of different arrangements which start and finish with the letter M. [2 pts]

M B B E E E M

$$\frac{6!}{2!3!} \checkmark$$

2. A committee of 6 animals is to be chosen from 8 different frogs and 5 different bunnies. In how many ways can the committee be selected if...

a) ...there must be more frogs than bunnies on the committee? (all frogs and no bunnies is ok) [2 pts]

all frogs, 5 frogs, 1 bunny, 4 frogs, 2 bunnies, 3

$$\binom{8}{6} + \binom{8}{5} \binom{5}{1} + \binom{8}{4} \binom{5}{2} \checkmark$$

b) ...the committee consists of 3 frogs and 3 bunnies, but two particular bunnies refuse to be on the committee together? [2 pts]

3 ways

$$\binom{8}{3} [\binom{5}{3} - 3] \checkmark$$

3. Find the number of positive integers greater than 6000 and less than 7000 which are divisible by 5, provided that no digit is to be repeated. [2 pts]

1 2 3 4 5 6 7 8 9 0

last is 5: 6 8 4 5

last is 0: 6 8 4 0

$8 + 4 = 12 \checkmark$

4. If  ${}_nP_r = 840$  and  ${}_nC_r = 35$ , then solve for  $r$  (your answer is a single number). [3 pts]

$\frac{n!}{(n-r)!} = 840$   $\frac{n!}{(n-r)!r!} = 35$   $\frac{840}{r!} = 35$   $r! = \frac{840}{35} = 24$   $r = 4 \checkmark$

$7 \cdot 3 \cdot 2 = 24$

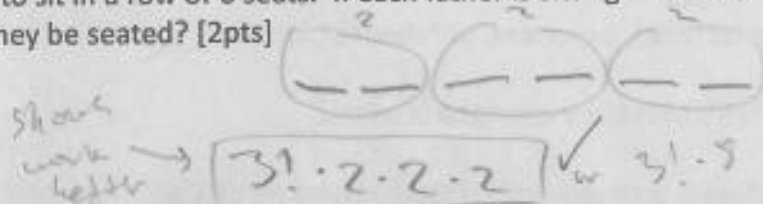
$20 \cdot 8 = 160$   $160 + 8 = 168$   $r! = \frac{168}{35} = 24$

5. My son's baseball team plays 11 games in the regular season. How many ways are there for the team to win 6 games, lose 3 and tie 2? (they only care about final standings, and not who they beat) [2pts]

wwwwwwlll++

$$\frac{11!}{6!3!2!} \checkmark$$

a) They want to sit in a row of 6 seats. If each father is sitting next to his own daughter, in how many ways can they be seated? [2pts]


$$\boxed{\frac{0}{3!} \frac{0}{3!} \frac{0}{3!}} \frac{0}{3!} \frac{0}{3!} \frac{0}{3!} \rightarrow \boxed{4! - 3!} \checkmark$$

seated? [2pts]  $\frac{\cancel{x} \cancel{x} \checkmark \cancel{\phantom{x}} \checkmark \cancel{\phantom{x}} \checkmark \cancel{\phantom{x}}}{\left(\frac{8}{2}\right) - 6!} \checkmark$   $\frac{8!}{2!}$  ← repeats & empty seats

$$q_{14}: \binom{16}{8} (-3b)^8 a^8 = \binom{16}{8} 3^8 a^8 b^8$$

$$b^{th} = \begin{pmatrix} 16 \\ 1 \end{pmatrix} (-36)^{n-2} a^2$$

$$\binom{16}{9} 3^9 b^9 a^2 - \binom{16}{9} (-3)^9 b^9 a^2 = 0$$

$$\begin{pmatrix} 16 \\ 8 \end{pmatrix} \begin{matrix} 1 & 8 & 8 \\ 3 & a & b \end{matrix} = \begin{pmatrix} 16 \\ 9 \end{pmatrix} \begin{matrix} 7 & 9 & 7 \\ 3 & b & a \end{matrix}$$

$$2 \cdot \binom{16}{7} a = \binom{16}{9} 3b$$

$$\frac{16! a}{8! 8!} = \frac{16! 36}{9! 7!} \quad 9! 7! 16! a = 16! 8! 8! 36$$

$$9a = 8 \cdot 36$$

$$\boxed{\frac{a}{b} = \frac{24}{9} = \frac{8}{3}} \quad \checkmark$$

MANGA

$$\frac{9!}{2!} = \frac{120}{2} = 60$$

$$- \sin \alpha \cos \beta \sin \gamma$$

-Shot with M

-the center

Final: AAGMN

Last: NMGAA

Start with N:N-----

$$\frac{4}{2} = 12 \text{ half}$$

FTK N: NAAGM

NAALM ✓