Analysis H 23/24
Hahn/ Hlasek/ Tantod
Unit 1: AtPS, Quiz 1
NO CALCULATORS

24

24 points

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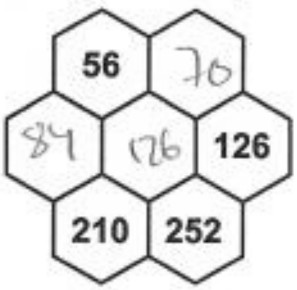
Period 6

Odd Number Triangle (Reminder: In the Odd Number Triangle, the row with [3 5] is the 2nd row.)

1. Write "true" or "false" for each statement. (1 pt each)
   a) The median of any row of the odd number triangle is a cube number. False
   b) The sum of all the terms in the first n rows of the odd number triangle is $\left(\frac{n(n+1)}{2}\right)^2$. True
   c) The sum of any two consecutive triangular numbers is a square number. True

Pascal's Triangle

2. The following flower is a portion of Pascal's Triangle. Find all the three missing numbers. (2 pts)



$252 - 126 = 126$ $\quad 126 - 56 = 70$ $\quad 210 - 126 = 84$

3. Simplify each expression below as a single term or a single binomial coefficient. (2 pts each)

   a) $\binom{20}{20} - \binom{20}{19} + \binom{20}{18} - \binom{20}{17} + \cdots + \binom{20}{2} - \binom{20}{1} + \binom{20}{0} =$ $\emptyset$

   $1 - 4 + 6 - 4 + 1 = 0$

   b) $\binom{k}{0} + \binom{k+1}{1} + \binom{k+2}{2} + \cdots + \binom{n}{n-k} = \binom{n+1}{n-k}$

Fibonacci Numbers

4. $F_n = (F_{61})^2 + (F_k)^2$. Solve for n and k. No proof or work shown is needed for this question. (1 pt)

   $k = 62$, $n = 123$ or $k = 60$, $n = 121$

   $60 + 2 = 120 + 1 = 121$

   $(F_x)^2 + (F_{x+1})^2 = F_{2x+1}$

5. Justify the following identity with a clear explanation. (3 pts)
$$2(F_4 + F_7 + F_{10} + F_{13} + F_{16} + F_{19}) = F_2 + F_3 + F_4 + F_5 + \cdots + F_{18} + F_{19}$$

$2F_4 + 2F_7 + 2F_{10} \cdots$

$F_4 = F_3 + F_2$

$F_7 = F_6 + F_5$

$F_2 + F_3 + F_4 + F_5 + F_6 + F_7 \cdots F_{19}$

$(F_4 + F_4) + (F_7 + F_7)$

Keep one of the pair and the other is the sum of the 2 below. This works because the Fibo. nums are all 3 appart and mult. by 2. Ends at F 19 because that's the last one.

10

Sequences and Series

6. Given that $a_2 = \frac{2}{49}$ and $a_6 = 98$, find the sum of the finite geometric series $\sum_{n=1}^{8} a_n$. Leave your answer as a numerical expression without sigma notation. (3 pts)

$98 = \frac{2}{49} \cdot r^4$

$r^4 = 49^2$

$r^4 = \sqrt[4]{49^2} = \sqrt[4]{7^4} = \pm 7$

$\frac{a_1(1-r^n)}{1-r}$

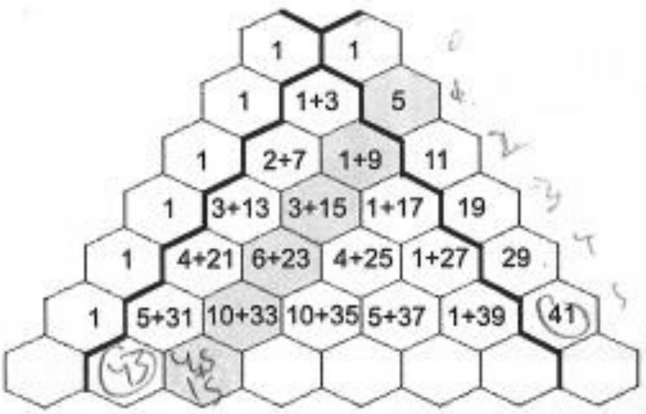
$\frac{2}{49} \cdot \frac{1}{\pm 7} = \pm \frac{2}{343}$

$50 \times 7 = 350 - 7 = 343$

$+ \quad \frac{\frac{2}{343}(1-7^8)}{-6}$ or $\frac{-\frac{2}{343}(1-7^8)}{8}$

The Fun Problem! ☺

For questions 7 and 8, refer to the array of numbers, created by overlapping Pascal's Triangle and the Odd Number Triangle by adding their terms.



7. The highlighted diagonal forms a sequence such that $a_1 = 5$, $a_2 = 1 + 9 = 10$, $a_3 = 3 + 15 = 18$, $a_4 = 6 + 23 = 29$, $a_5 = 10 + 33 = 43$. $(5+10)+45$

a) Find a_6. (1 pt)

$45 + 15 = 60$

b) Find a formula for a_n in terms of n. (3 pts)

5, 10, 18, 29, 43, 60

5 8 11 14 17

3 3 3 3

$\frac{3 \cdot 25 + 5 + 6}{2}$ $\quad 75 + 11 = \frac{86}{2} = 43$ ✓

$5 + \left(\frac{5+5+3(4)}{2}\right)5$

$\frac{22}{2} \cdot 5 + 5$

$55 + 5 = 60$ ✓

$5 + \left(\frac{5+5+3(n-2)}{2}\right)(n-1)$

$5 + \left(\frac{10+3n-6}{2}\right)(n-1)$ $\quad 5 + \left(\frac{3n+4}{2}\right)(n-1)$ $\quad \frac{10}{2} + \left(\frac{3n^2+4n-3n-4}{2}\right)$

$\boxed{\frac{3n^2+n+6}{2}}$

8. The sum of row 0 is $1 + 1 = 2$. The sum of row 1 is $1 + 4 + 5 = 10$. The sum of row 2 is $1 + 9 + 10 + 11 = 31$.

a) Find the sum of the 4-th row. (1 pt)

29, 29, 29, 29, 25 $= 141$

b) Find a formula for the sum of the n-th row in terms of n. (3 pts)

1 1

3 5 8

7 9 11 27

13 15 17 19 64

n^3

1 1 2

1 2 1 4

1 3 3 1 8

1 4 6 4 1 16

2^n

$n^3 + 2^n$

$2^3 + 2^2 = 8 + \cancel{4} = 12$

$(n+1)^3 + 2^n$

$5^3 + 2^4 = 125 + 16 = 141$ ✓

$3^3 + 2^2 = 27 + 4 = 31$ ✓

F0