Analysis Honors 23/24 Serial converger: ______________________

Hahn / Hlasek / Tantod Period: _____

Unit 8 Biggish Quiz: Sequences and Series ________

No calculators. 50 pts

Trigonometric functions have arguments in radians.

<u>Part I: Sequences</u> (20 pts)

Question #1-3 are multiple choice. Circle one best answer. [1pt each]

1. Sequence $\{n^p\}$ converges if and only if

   a) $p \geq 0$ b) $p \geq 1$ c) $p \leq 0$ d) $p < -1$ or $p \geq 1$ e) $-1 < p \leq 1$

2. Which of the sequences below is everywhere increasing and bounded above?

   a) $\{\sin n\}$ b) $\left\{\sin\left(\frac{\pi n}{2}\right)\right\}$ c) $\{n^2 - 3n + 2\}$ d) $\left\{-\frac{1}{2^n}\right\}$ e) $\left\{\frac{n}{n-1}\right\}$

3. $\left\{\frac{7}{r^{2n}}\right\}$ converges if and only if

   a) $r > 0$ b) $r < 0$ c) $r \geq 1$ d) $r > 1$ e) $r \leq 1$ f) $r < 1$

4. Answer True or False for each statement below. [1 pt each]

   a) If a sequence converges, it must have an upper bound. __________

   b) If $a_n < b_n$ for all natural n and $\{b_n\}$ converges, then $\{a_n\}$, must converge. __________

   c) If a sequence $\{a_n\}$ is divergent and satisfies $a_n \geq 0$ for all natural n, it must increase without bound. ___________

5. Find the limit of each sequence or state that it diverges. No justification needed. [2 pts each]

   a) $\left\{\frac{3n}{\sqrt{4n^2+1}}\right\}$ b) $\{a_n\}$, where $a_n = 5\sqrt{a_{n-1}}$ and $a_1 = 5$ c) $\left\{\frac{n^n}{n!}\right\}$

6. Consider the sequence $\{b_n\}$, where $b_n = \begin{cases} 3 & \text{if n is odd} \\ \frac{6}{2n+1} & \text{if n is even} \end{cases}$.

State the value to which the sequence converges, or state that the sequence diverges. CLEARLY justify your answer. If your work is correct but is difficult to interpret, you may not receive full credit. [4 pts]

The **value** to which the sequence converges (or say “diverges”): ___________

Show your work here:

7. For the sequence $\left\{\frac{6^n}{(2n-1)!}\right\}$, state the value to which the sequence converges, or state that the sequence diverges. CLEARLY justify your answer with a proof. For your proof, you MUST use one of the following tests: the squeeze theorem OR the big theorem (bounded above/below and always increasing/decreasing theorem) OR the comparison principle. If your work is correct but is difficult to interpret, you may not receive full credit. [4 pts]

Test/Principle used: _________________________

The **value** to which the sequence converges (or say “diverges”): ___________

Show your work here:

Part II: Series (30 pts)

8. Consider a sequence $\{a_n\}$ and its corresponding partial sums $s_k = \sum_{k=1}^{n} a_k$. Given that $\sum_{n=1}^{\infty} a_n = 1$, determine the following limits. No explanation required. [1 pt each]

   a) $\lim_{n\to\infty} a_n =$ ____ b) $\lim_{n\to\infty} s_n =$ ____

9. Consider a sequence $\{b_n\}$ and its corresponding series $\sum_{n=1}^{\infty} b_n$. For each statement below, give an example that satisfies the given requirements, or state that none exists. [2pts each]

   a) $\{b_n\}$ is everywhere increasing and $\sum_{n=1}^{\infty} b_n$ converges. $b_n =$ ________________

   b) $\{b_n\}$ alternates and $\sum_{n=1}^{\infty} b_n$ diverges. $b_n =$ ________________

   c) $\{b_n\}$ converges and $\sum_{n=1}^{\infty} b_n$ converges to 4. $b_n =$ ________________

10. For each series below, write “C” if it converges and “D” if it diverges. No justification needed. [2 pts each]

    $a)\ \sum_{n=1}^{\infty} \frac{1}{\ln(n+1)}$ $b)\ \sum_{n=1}^{\infty} \frac{2n^2+3n+1}{\sqrt{9n^4+n+7}}$ $c)\ \sum_{n=1}^{\infty} \frac{3^n+1}{2^{2n}}$

    ____________ ____________ ____________

11. For each series below, write a clear proof to show the convergence or divergence of the series. Indicate if the series converges or diverges and name the test used. If you choose to use more than one test for a series, state names of all the tests used. [5 pts each]
<u>Important: for the two series below, you MAY NOT use the same test twice!</u>
(If you use the same test for both series, you will lose 3 points.)

a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^n}{(n+1)!}$

Circle one: converges / diverges

Test used: ________________________________

b) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^2\sqrt{n}}$

Circle one: converges / diverges

Test used: ________________________________

12. Consider the region $\mathcal{R}$ bounded by the function $f(x) = \frac{x^3}{3}$ and the x-axis in the interval [0,3] as shown on Diagram 1 below.

Diagram 1: region $\mathcal{R}$

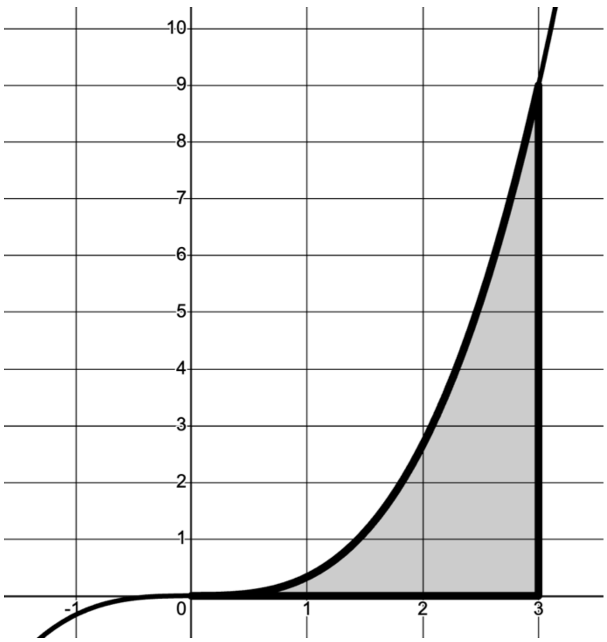
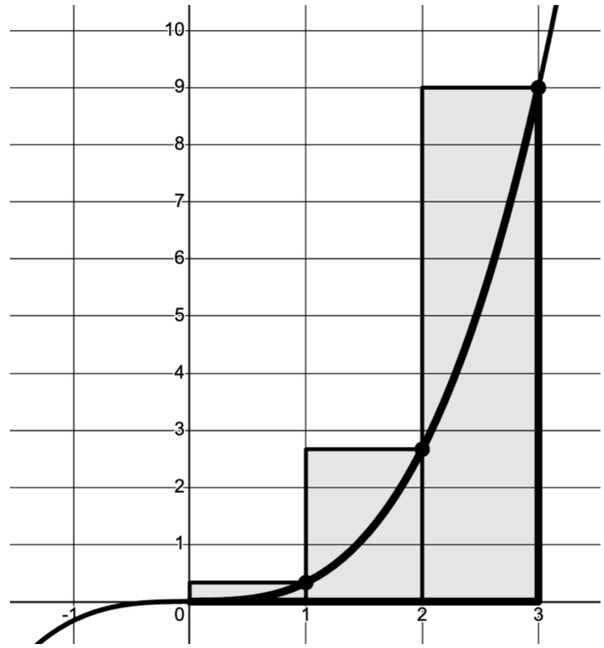


Diagram 2: area A_1



a) Find the area A_1 of the region shaded in Diagram 2 that consists of three rectangles with width 1 whose upper right corners lie on the graph of $y = f(x)$. [1 pt]

b) Circle one: Area of the region $\mathcal{R}$ is **smaller** / **greater** than the area A_1. [1 pt]

c) Find an algebraic expression in terms of n for the area A_n of the region that consists of $3n$ rectangles with width $\frac{1}{n}$ whose upper right corners lie on the graph of $y = f(x)$. [3 pts]
Potentially helpful formulas:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

d) Determine the area of region $\mathcal{R}$ by finding $\lim_{n\to\infty} A_n$. [1pt]