Analysis H 2023-2024
Hahn / Hlasek / Tantod
Unit 2 (Probability) Quiz 2
NO CALCULATORS

22
27 pts

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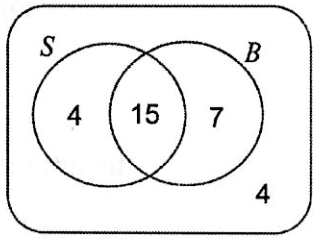
will probably do well on this quiz
Date: 9/22/23 Period: 6



For Problems 1 and 2: The Venn Diagram on the right shows responses to the following survey question: "Do you eat pancakes with syrup or bacon?" S represents syrup and B represents bacon. The survey had 30 respondents.

1. Which of the following probabilities are equal to $\frac{1}{2}$?
   Circle **all** the apply. *(3 pts total)*

   a) $P(S \cap B)$ $\frac{15}{30}$ b) $P(S \cup B) = \frac{26}{30}$ c) $P(S' \cap B) = \frac{8}{30}$

   d) $P(S' \cup B')$ $\frac{15}{30}$ e) $P(B \mid S') = \frac{7}{11}$ f) $P(S \mid B') = \frac{4}{8}$

2. Answer true or false for each statement below. *(1 pt each)*

   a) S and B are mutually exclusive. false has an intersection

   b) S and B are independent. false $P(S) \neq P(S \mid B)$ vice versa.

3. I draw a hand of 4 cards from a standard deck of 52 cards. What is the probability that I have 2 Queens, given that I have exactly 2 Aces? Leave your answer in factorial, exponent, and/or choose number form. *(3pts)*

Probability = $P(2Q \mid 2A) = \frac{P(2Q \cap 2A)}{P(2A)} = \frac{\binom{13}{1}\binom{4}{2}\binom{13}{1}\binom{4}{2}}{\binom{13}{1}\binom{4}{2}\binom{13}{1}\binom{4}{2}} = \binom{13}{1}\binom{4}{2}$

2 Aces won't make a difference in $P(2Q)$ → Independent event
↳ $P(2Q \mid 2A) = P(2Q) = \binom{13}{1}\binom{4}{2}$
↑ chose Q, chose 2 suits

4. A pack of Starburst contains 12 pieces of chewy candy: 2 yellow, 4 pink, and 6 red. Firstly, you choose one candy at random and eat it. Then, you choose 2nd candy at random and eat it. Find the following probabilities as completely simplified fractions. *(2pts each)*

   a) P(both pieces of candy are yellow)

   $\frac{2}{132} = \frac{1}{66}$

   b) P(2nd candy is pink)

   $\frac{8+12+24}{132} = \frac{44}{132} = \frac{22}{66} = \frac{2}{6} = \frac{1}{3}$

   c) P(1st candy is pink | 2nd candy is pink)

   $\frac{P(A \cap B)}{P(B)} = \frac{\frac{12}{132}}{\frac{1}{3}} = \frac{\frac{1}{11}}{\frac{1}{3}} = \frac{3}{11}$

$\frac{3}{5}$ hit

5. A (blindfolded) marksman hits the target 3 times out of 5 times. If he fires 4 shots, find the following probabilities. Leave your answers in factorial, exponent, and/or choose number form. *(2 pts each)*

   a) P(more than 2 hits)

   $1 - \binom{4}{0}\left(\frac{3}{5}\right)^0\left(\frac{2}{5}\right)^4 - \binom{4}{1}\left(\frac{3}{5}\right)^1\left(\frac{2}{5}\right)^3$ = $1 - \left(\frac{2}{5}\right)^4 - \binom{4}{1}\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^3$ −1

   All hit 0 hit 1 hit

   b) P(at least 3 misses)

   $\binom{4}{0}\left(\frac{2}{5}\right)^0\left(\frac{2}{5}\right)^4 + \binom{4}{1}\left(\frac{3}{5}\right)^1\left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)^4 + \binom{4}{1}\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^3$

   4 miss 3 miss

6. A fair coin is tossed n times, where n is a positive integer. The probability that a head occurred 10 times is the same as the probability that a head occurred 8 times. Find the value of n. Your answer should be a single integer. *(3 pts)*

   $\binom{n}{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^{n-10} = \binom{n}{8}\left(\frac{1}{2}\right)^{8}\left(\frac{1}{2}\right)^{n-8}$

   $\binom{n}{10}\left(\frac{1}{2}\right)^{n} = \binom{n}{8}\left(\frac{1}{2}\right)^{n-8}$

   $\binom{n}{10} = \binom{n}{8}$

   $\binom{n}{10} = \frac{n!}{10!(n-10)!} = \frac{n!}{8!(n-8)!} = \binom{n}{8}$

   n = 18

7. Gunn Casino offers a game where two fair 6-sided dice are rolled and the numbers that were rolled are multiplied. If the product is even, you receive $2. If the product is one, you receive $9. It costs $1.50 to play this game. What is the expected value of playing this game? Explain why you would or would not play. *(3 pts)*

×	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

   even? +2
   = 1? +9
   −1.50 each

   $\frac{1}{36}(9) + \frac{27}{36}(2) - 1.50 = \frac{9}{36} + \frac{54}{36} - 1.50$

   =1 evens any

   $= \frac{63}{36} - 1.50$

   $= \frac{7}{4} - \frac{3}{2}$

   $= \frac{7}{4} - \frac{6}{4} = \frac{1}{4} = 0.25$ win per play

   I would play because, on average, I expect to win 25 cents, each play.

8. A spinner wheel with integers from 1 to n is spun once. If each number is equally likely to be the outcome, find the expected value in terms of n. Show the work that leads to your answer. *(3 pts)*

   $E[X] = n \cdot p$ (times, probability)

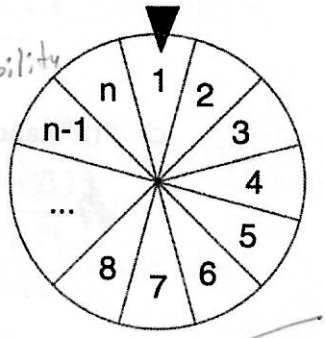
   $= 1 \cdot \frac{1}{n}$

   $= \frac{1}{n}$ −2

   $E[X]$ = times want · probability

   $= 1 \cdot \frac{1}{n}$

   $= \frac{1}{n}$



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