Analysis H – Hahn / Hlasek / Tantod
AtPS Quiz 2, 2023-24
NO CALCULATORS

25 / 25 points

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Period: 6

1. Simplify each expression to a single Fibonacci number. Show your work to receive full credit. [3 pts each]

a) $2(F_1 + F_4 + F_7 + F_{10} + \cdots + F_{334}) =$

$2F_1 + 2F_4 + 2F_7 + 2F_{10} + \dots + 2F_{334}$

$\cancel{F_2} - F_0 + \cancel{F_3} - \cancel{F_2} + F_5 - \cancel{F_3} + F_6 - \cancel{F_5} \dots$

$+ \cancel{F_{335}} - \cancel{F_{333}} + F_{336} - \cancel{F_{335}}$

$F_{336} - F_0$

$\boxed{F_{336}}$ ✓

b) $F_{17} + 4F_{18} + 6F_{19} + 4F_{20} + F_{21} =$

$3F_{18} + 7F_{19} + 4F_{20} + F_{21}$

$4F_{19} + 7F_{20} + F_{21}$

$3F_{20} + 5F_{21}$

$2F_{21} + 3F_{22}$

$F_{22} + 2F_{23}$

$F_{23} + F_{24} = \boxed{F_{25}}$ ✓

2. Evaluate ("evaluate" means "give the value of". Your answer should be a single number). Show the work that you used to arrive at your answer. [3 pts]

$$\sum_{5}^{\infty} 1024\left(\frac{1}{2}\right)^n$$

$\frac{a_1}{1-r} \to \frac{a_5}{1-r}$ 512, 256, 128, 64, 32

$1024\left(\frac{1}{2}\right)^5$

$\frac{32}{1-\frac{1}{2}} = \frac{32}{\frac{1}{2}}$ 64 ✓

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

32
+32
+64
+(illegible)
1024

−0 3. Write this in Sigma Notation: [3 pts] $72 + 66 + 60 + 54 + \ldots - 594 - 600$

$$\sum_{n=1}^{113} [72 - 6(n-1)]$$

600
+72
672

$\frac{672}{6} = 112$ +1

112 × 6 = 672

112 × 6 = 672 ✓

4. Solve for x in terms of n and k, simplified as much as possible. Leave your answer in factored form (no need to multiply it out), and you can also leave factorials in your answer, if necessary. [2 pts]

$$\frac{(n-1)!}{(k-1)!(n-k)!} \cdot \frac{n!}{(k+1)!(n-k-1)!} = \frac{(n+1)!}{k!(n-k+1)!} \cdot \frac{(n-1)!}{k!(n-k-1)!} \cdot x$$

$\frac{(n-1)!}{(k-1)!(n-k)!} \cdot \frac{n!}{(k+1)!(n-k-1)!} \cdot \frac{k!(n-k+1)!}{(n+1)!} \cdot \frac{k!(n-k-1)!}{(n-1)!} = x$

$\frac{(n-1)!}{(n-1)!} = 1$ $\frac{(n-k-1)!}{(n-k-1)!} = 1$

$\frac{k!}{(k+1)!} = \frac{k!}{k! \cdot k+1} = \frac{1}{k+1}$

$\frac{n!}{(n+1)!} = \frac{n!}{n! \cdot n+1} = \frac{1}{n+1}$ $\frac{(n-k+1)!}{(n-k)!} = \frac{n-k+1 \cdot (n-k)!}{(n-k)!} = n-k+1$

$\frac{k!}{(k-1)!} = \frac{(k-1)! \cdot k}{(k-1)!} = k$

$$\frac{k(n-k+1)}{(k+1)(n+1)} = x$$ ✓

$k \cdot (n-k+1) \cdot \frac{1}{k+1} \cdot \frac{1}{n+1} = \frac{k(n-k+1)}{(k+1)(n+1)}$

−0

1 (8) 36

5. Consider the summation: $S = \frac{4}{5!} + \frac{5}{6!} + \frac{6}{7!} + \cdots + \frac{102}{103!}$

We can use **telescoping** to write S as a compact expression if we replace the numerators like this:

$$S = \frac{5-1}{5!} + \frac{6-1}{6!} + \frac{7-1}{7!} + \cdots + \frac{103-1}{103!}$$

Continue simplifying to write S as a compact expression. [3 pts]

$$\frac{5}{5!} - \frac{1}{5!} + \frac{6}{6!} - \frac{1}{6!} + \frac{7}{7!} - \frac{1}{7!} \cdots + \frac{103}{103!} - \frac{1}{103!}$$

$$\frac{1}{4!} - \cancel{\frac{1}{5!}} + \cancel{\frac{1}{5!}} - \cancel{\frac{1}{6!}} + \cancel{\frac{1}{6!}} - \cancel{\frac{1}{7!}} \cdots + \cancel{\frac{1}{102!}} - \frac{1}{103!}$$

$$\frac{1}{4!} - \frac{1}{103!}$$ ✓

6. Evaluate each (each answer should be a single number). [1 pt each] $\frac{(n)(n-1)(n-2)\ldots(n-k+1)}{k!}$

a) $\binom{20}{3}$

$\frac{20!}{3!(17!)}$

$\frac{20 \cdot 19 \cdot 18}{6}$

380 · 3

1140 ✓

b) $\binom{3}{-4}$

0 ✓

c) $\binom{-2}{7}$

$\frac{-2 \cdot -3 \cdot -4 \cdot -5 \cdot -6 \cdot -7 \cdot -8}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}$

$-1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 \cdot -1 \cdot -1$

$1 \cdot 1 \cdot 1 \cdot -1$

-1 -8 ✓

d) $\binom{12}{15}$ $15 > 12$

0 ✓ $\cancel{\frac{12!}{15!(-3)!}}$

all positive data

7. Prove using Mathematical Induction: [4 pts] "$11^n - 6$ is divisible by 5 for all values of $n > 0$"

$n=1$ case: $11^1 - 6 = 5$ ✓ 5 divisible by 5

assume: $11^k - 6$ is divisible by 5

prove: $11^{k+1} - 6$ is divisible by 5

$11 \cdot 11^k - 6$

$11^k - 6 + 10 \cdot 11^k$

$\underbrace{(11^k - 6)}_{\text{divisible by 5}} + \underbrace{(5 \cdot 2 \cdot 11^k)}_{\text{divisible by 5}}$

$\boxed{\text{QED}}$

Sum of geom

$\frac{a_1(1-r^n)}{1-r}$

$\cancel{\frac{n(n+1)}{2}}$

$\frac{n(\text{first} + \text{last})}{2}$

first · $r^{\text{\# of terms}}$

1—0