Group Theory Quiz 1   30   I am not going to snap over this quiz!!! Michi Tanaka
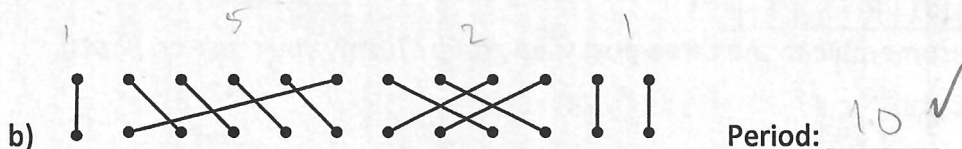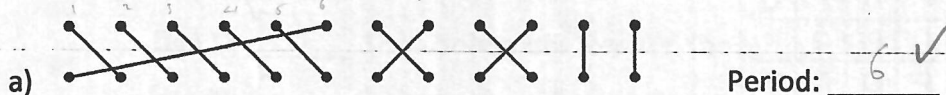Analysis 23/24   Period 6
Hahn / Hlasek / Tantod   30   No calculator. You **can** use a flip triangle for this quiz.

1. Below are some of the elements of the 12-post snap group. Write the period of each.

   [2 pts each]

   a)   Period: __6__ ✓

   b)   Period: __10__ ✓

2. Draw an element of the 12-post snap group with period 60 or state that there is no such element. *[2 pts]*

   $3,4,5 = 60$

    ✓
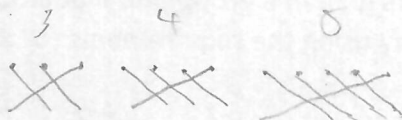
   $$\begin{array}{c|c} 60 & \\ \hline \cdot & 60 \\ 2 & 30 \\ 3 & 20 \\ 4 & 15 \\ 5 & 12 \\ 6 & 10 \end{array}$$

3. What is the **maximum** period of an element in the 27-post snap group? Show how you got your answer. *[3 pts]*

   Sum: 27
   relatively prime
   lcm_max

   $lcm(2,5,7,13) = 910$

   $lcm(4,5,7,11) = \boxed{1540}$ ✓

   $4+5+7+11 = 27$

   $$\frac{140}{11} \quad 1540$$

   $$\frac{84}{13}$$

   $2,3,4,5,13$

4. Each element of D_3 can be generated using only elements **F** and **R** defined in the table below. Represent each of the remaining three elements of D_3 <u>using as few operations as possible</u>. Some elements will require you to use both **F** and **R**, others can be represented using only one of them. *[3 pts]*

F²	RF ✓	FR ✓	F	R² ✓	R

5. Group H is defined by the group table below.

	P	Q	R	S	T	U
P	P	Q	R	S	T	U
Q	Q	R	S	T	U	P
R	R	S	T	U	P	Q
S	S	T	U	P	Q	R
T	T	U	P	Q	R	S
U	U	P	Q	R	S	T

a) What is the identity? ___P___ [1 pt]

b) What is the order of H? ___6___ [1 pt]

c) What is the inverse of T? ___R___ [1 pt]

d) What is the period of S? ___2___ [2 pts]

e) What is the period of U? ___6___ [2 pts]

f) Is group M̶ isomorphic to the three-post snap group? Justify your answer. [2 pts]
   H

order: same 6   periods: snap, 1,2,3
entries: same 36   H: ...6 ⟵ not same
                                not Isomorphic ✓

6. Is the set of numbers {0, i, -i} a group under addition? If yes, state which element is the identity. If not, justify using the requirements for a group. [2 pts]
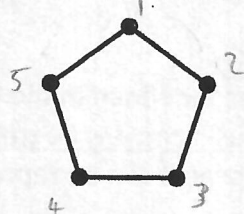
No: No closure $i + i = 2i \in$ not in group. ✓

7. Consider the rotation group for regular pentagon where the element shown below is the identity element.
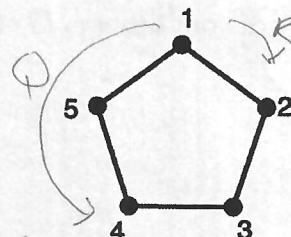Let **R** be the operation "rotate 72° clockwise".
Let **Q** be the operation "rotate 144° counterclockwise".
   a) Draw the element represented by **RQR**. [2 pts]

-144°
72° 72° $2(72) - 144 = 0°$
            ⟲ I

b) What is the order of the group generated by **R** and **Q**? Justify your answer. [1 pt]

Order 5. from R to R^4 is the most operations possible before repeating ✓

c) What is the order of the group generated by **Q** alone? [1 pt]

order 5. Returns to I after Q^4 ✓

8. Consider group G = {all real numbers that are not equal to -1} under binary operation $\otimes$ defined as $a \otimes b = a + b + ab$.
   a) What is the identity in this group? [1 pt]

0 ✓

b) Find the inverse of 3. Show your work. [2 pts]

$3 \otimes b = 3 + b + 3b$ $3 \otimes b = 0$

$0 = 3 + b + 3b$
$-3 = b + 3b$ $-3 = 4b$ $\boxed{b = \frac{-3}{4}}$ ✓
$-3 = (1+3) b$

$3 + \frac{-3}{4} + \frac{-3}{4} 3 = 3 - \frac{3}{4} - \frac{9}{4}$
$= \frac{12}{4} - \frac{3}{4} - \frac{9}{4}$
$= 0$

-0