Analysis H – Hahn / Hlasek / Tantod   56   How should Andersen Tahwiki handle the too-bright limit? Dim it.
Biggerish Quiz on Ch 9: Calculus   Period: 6
NO CALCULATORS   56 pts

1. A certain function has the values as shown in the table below.

x	3	5	8	11
f(x)	30	20	5	2

a) What is the average rate of change of the function for the x-interval [3,11]? [2 pts]

$$\frac{f(11)-f(3)}{11-3} = \frac{2-30}{8} = \frac{-28}{8} = \boxed{\frac{-7}{2}} \checkmark$$

b) Use a backwards (left) difference quotient to approximate the IROC of f(x) at x = 5. [2 pts]

$$\frac{f(5)-f(3)}{5-3} = \frac{20-30}{2} = \frac{-10}{2} = \boxed{-5} \checkmark$$

c) Consider the statement, "$f(c) = 15$ for some c value: 5 < c < 8". Is this statement TRUE or FALSE? Provide mathematical justification (with appropriate vocabulary) for your answer. [2 pts]

False because it is not given the function is continuous so the statement cannot be upheld. There could be no valid x values from 5 to 8. ✓

d) Suppose f(x) modeled the acceleration of a particle (in meters per second²) over the time interval of x (in seconds). Use 3 trapezoids to approximate the definite integral of f(x) over [3,11]. Give your answer as a single number, and include units in your answer. [4 pts]

$$\text{Area} = \frac{(B_1+B_2)h}{2}$$

$$\frac{m}{s^2} \cdot s = \frac{m}{s}$$

$$\frac{(30+20)2}{2} + \frac{(20+5)3}{2} + \frac{(5+2)3}{2} =$$

$$50 + 37.5 + 10.5 =$$

$$50 + 48 = 98 \qquad \boxed{98\ \frac{\text{meters}}{\text{second}}} \checkmark$$

/–0

2. Consider the function $f(x)=\frac{1}{x}$. Use the Formal Definition of Derivative at a point to find $f'(4)$. In order to receive full credit, you need to show all your work and use proper notation throughout. [3 pts]

$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

$$f'(4)=\lim_{x\to 4}\frac{\frac{1}{x}-\frac{1}{4}}{x-4}=\lim_{x\to 4}\frac{4-x}{4x(x-4)}\cdot\frac{-1}{-1}=\lim_{x\to 4}\frac{\cancel{x-4}}{-4x(\cancel{x-4})}=\lim_{x\to 4}\frac{1}{-4x}=\boxed{\frac{-1}{16}}$$

✓

3. Consider the function $g(x)=-2x^3+5x^2+9$. Use the Formal Definition of Derivative to find ~~$g'(4)$~~ $g'(x)$. In order to receive full credit, you need to show all your work and use proper notation throughout. [3 pts]

$$g'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$-2(x+h)^3+5(x+h)^2+9$

$(x^2+2xh+h^2)(x+h)$

$h^3+x^3+x^2h+2x^2h+2xh^2+h^2x$

$h^3+x^3+3x^2h+3xh^2$

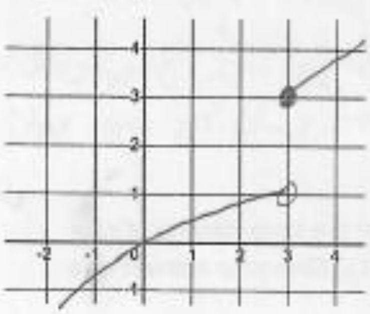
$$=\lim_{h\to 0}\frac{-2h^3-2x^3-6x^2h-6xh^2+5x^2+10xh+5h^2+9+2x^3-5x^2-9}{h}$$

$$\lim_{h\to 0}\frac{-2h^3-6x^2h-6xh^2+10xh+5h^2}{h}=\lim_{h\to 0}-2h^2-6x^2-6xh+10x+5h=\boxed{-6x^2+10x}$$

✓ $[g'(x)]$

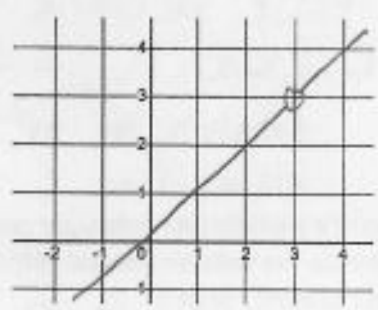
4. For each of the following, draw a function to fit the description. [1 pt each]

a) f(3) exists, but the limit as x approaches 3 does not exist.



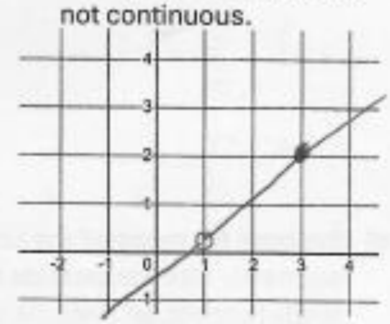
✓

b) there is a limit as x approaches 3, but f(3) does not exist.



✓

c) there is a limit as x approaches 3 and f(3) exists, but the function is not continuous.



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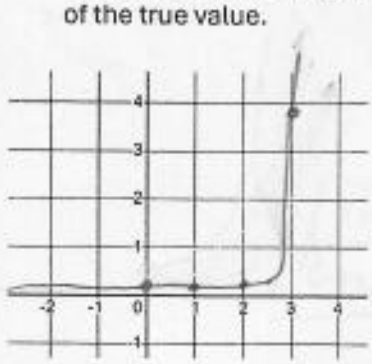
d) Using trapezoids to find the definite integral over the interval [0, 3] would result in an overestimate of the true value.



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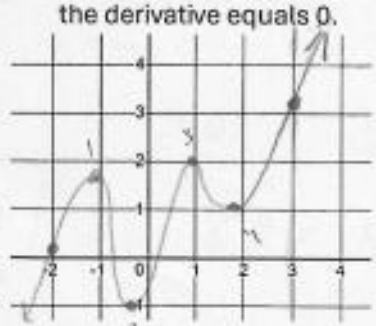
e) The derivative exists for all values in the interval [-2, 3], and there are exactly 4 values in the interval [-2, 3] for which the derivative equals 0.

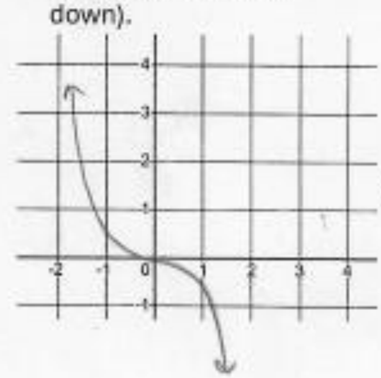


✓

f) The derivative of the function is a parabola whose vertex is its maximum value (the parabola is pointing down).

$-x^3$

$-3x^2$



✓

-0

5. Fill in the blanks to complete the Definition of a Limit. [3 pts total]

$\lim_{x \to c} f(x) = L$ if and only if for all values of positive ε, there exists a δ such that if X is within δ of C, but not equal to C, then $f(x)$ will be within ε of L. ✓

6. Consider the function $f(x) = \sqrt{x-2} + 5$.

a) $\lim_{x \to 18} f(x) =$ 9 [1 pt] ✓

b) Use delta, epsilon, and the limit definition to prove your answer from part (a). Your answer must include a conclusion statement. [4 pts]

$9-\varepsilon < \sqrt{x-2} + 5 < 9+\varepsilon$

$4-\varepsilon < \sqrt{x-2} < 4+\varepsilon$

$(4-\varepsilon)^2 + 2 < x < (4+\varepsilon)^2 + 2$

$18 - 8\varepsilon + \varepsilon^2 \qquad 18 + 8\varepsilon + \varepsilon^2$

$18 + 8\varepsilon + \varepsilon^2 - 18 = 8\varepsilon + \varepsilon^2$

$18 - (18 - 8\varepsilon + \varepsilon^2) = 8\varepsilon - \varepsilon^2$ ← smaller

$8\varepsilon - \varepsilon^2 = \delta$

δ

For any positive # ε, if x is with $8\varepsilon - \varepsilon^2$ of 18, $f(x)$ is within ε of 9 ✓

$c \qquad \sqrt{x-2} + 5 \qquad L$

7. For a certain function $h(x)$, I can find a solution for $h(x) = R$, where R is any arbitrarily large number, by plugging in an x-value that is sufficiently close to -7. This proves that (fill in the boxes): [2 pts]

$$\lim_{\boxed{x} \to \boxed{-7}} h(x) = \boxed{\infty}$$ ✓ ✓

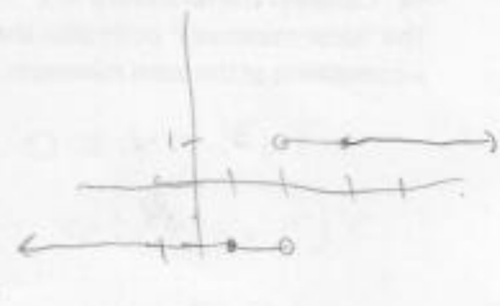
8. Evaluate each limit, or state that the limit does not exist. [2 pts each]

a) $\lim_{x \to 1} \frac{x^2+5x-6}{x-1} = \lim_{x \to 1} \frac{(x+6)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x \to 1} x+6 = \boxed{7}$ ✓

$\frac{x-5}{(x^2-25)(x^2+25)} = \frac{x-5}{(x-5)(x+5)(x^2+25)}$

b) $\lim_{x \to 5} \frac{x-5}{x^4-5^4} = \lim_{x \to 5} \frac{(x-5)}{(x-5)(x+5)(x^2+25)} = \lim_{x \to 5} \frac{1}{(x+5)(x^2+25)} = \boxed{\frac{1}{500}}$ ✓

c) $\lim_{x \to 2} \frac{x-2}{|x-2|} = \boxed{\text{DNE}}$ ✓



$\frac{-2}{-8} = \frac{1}{4}$

d) $\lim_{x \to 2} \frac{x^2-5x+4}{x^2-2x-8} = \lim_{x \to 2} \frac{(x-4)(x-1)}{(x+2)(x-4)} = \lim_{x \to 2} \frac{x-1}{x+2} = \boxed{\frac{1}{4}}$ ✓

$\frac{4-10+4}{4-4-8} = \frac{-2}{-8} = \frac{1}{4}$

/-0

9. Find the derivative of each function using any method. [2 pts each]

a) $f(x) = 7x^3 - 5x^2 + 10$

$\boxed{f'(x) = 21x^2 - 10x}$ ✓

b) $g(x) = 8x^{(2/3)} - 10\sqrt{x} + \frac{4}{x^5}$

$8x^{\frac{2}{3}} - 10x^{\frac{1}{2}} + 4x^{-5}$

$\boxed{g'(x) = \frac{16}{3}x^{-\frac{1}{3}} - 5x^{-\frac{1}{2}} - 20x^{-6}}$ ✓

c) $h(x) = e^7 + \pi^{-3}$

$\boxed{h'(x) = 0}$ ✓

11. Consider the function $f(x) = \frac{3}{2}x^2 - 5x + 4$ $\quad f'(x) = 3x - 5 \quad f(3) = 13.5 - 15 + 4$

$f(3) = 2.5$

a) Find the equation of the line tangent to f(x) at x = 3. [3 pts]

$y - f(3) = f'(3)(x - 3)$

$f'(3) = 4$

$\boxed{y - 2.5 = 4(x - 3)}$ ✓

b) There are two lines tangent to f(x) that also pass through the point (4, 2). Find the slope of each of the lines. [4 pts]

$3x - 5 = \frac{\frac{3}{2}x^2 - 5x + 4 - 2}{x - 4}$

$f'(6) = \boxed{13}$ ✓

$f'(2) = \boxed{1}$

$3x^2 - 5x - 12x + 20 = \frac{3}{2}x^2 - 5x + 2$

$6x^2 - 34x + 40 = 3x^2 - 10x + 4$

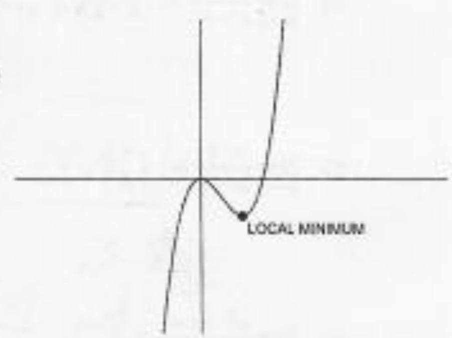
$3x^2 - 24x + 36 = 0$

$(x-6)(x-2) = 0$

$x^2 - 8x + 12 = 0$

$x = 6, 2$

12. Consider the function $y = x^3 - 2x^2$, whose graph is shown on the right. The "local minimum" point also shown on the graph. Use the Power Rule to find the x-coordinate of the local minimum. [3 pts]



$3x^2 - 4x = 0$

$x(3x - 4) = 0$

$x = \frac{4}{3}, 0$

$\boxed{x = \frac{4}{3}}$ ✓

-0