Analysis H - Hahn / Hlasek / Tantod GAtM Exam, 2023-2024 No Calculators Groupie: _____

Period: _____

1. For a certain complex number z, Re(z)>0 and Im(z)>0. Answer "Always", "Sometimes" or "Never" for each of the following statements. [1 pts each]

80 points

- a) Re(z²) > 0 _____
- b) Im(z²) > 0 _____
- c) Re(z 2i) > 0 _____
- d) Im(z 2i) > 0 _____
- e) Re(*z*) > 0 _____
- f) Im(*z*) > 0 _____

2. For each set listed below, give the number of elements in the set, or write "C" if the set is countably infinite, or "UNC" if the set is uncountably infinite. [2 pts each]

- a) Points on a line segment _____
- b) Rational Numbers excluding 0 _____
- c) Prime Numbers _____
- d) Elements in the Cantor Set _____
- e) Elements in the rotation/reflection group of a cube _____
- f) Elements in the 6-post snap group _____
- g) Elements in the 4-post snap group that have a period equal to 2 _____
- h) Elements in the group generated by "rotate 5 degrees" and "rotate -5 degrees"
- i) Elements in the group generated by "rotate 5 radians" and "rotate -5 radians" _____
- j) Complex numbers in the form a + bi, where a and b are integers _____



3. One way to map the points within a 2D square (let's call this "Set A") onto the points on a line segment ("Set B") is to first create a coordinate system for each, as shown above. Then take the (x,y) coordinate of a point in SET A and <u>alternate the digits</u> to create a corresponding point in SET B.

a) Fill in the table of values to complete the mapping as described above. The first is done for you. [2 pts each]

SET A point	SET B point
(0.357, 0.429)	0.345279
(0.123, 0.456)	
	0.4758
(0.3, 0.987)	

b) Andy doesn't like the mapping method used in the table in part (a), and would rather just add the x and y-values of the Set A point to get a Set B value: (0.357, 0.429) → 0.786. Explain, using at least one counterexample, why this would <u>not</u> be a one-to-one correspondence between Set A and Set B. [3 pts]

c) Beth also doesn't like the mapping method used in the table in part (a), and wants to map the points from Set A to Set B this way: $(0.357, 0.429) \rightarrow 0.357429$. Explain, using at least one counterexample, why this would **not** be a one-to-one correspondence between Set A and Set B. [3 pts]

4. Is there a one-to-one correspondence between the set of complex numbers and the set of 2x2 transformation matrices? If yes, describe the one-to-one correspondence and include a non-zero example. If no, give an example of an element in one set that does not have a corresponding element in the other set. [3 pts]

- 5. Consider the complex numbers x = -2 + 3i, y = -3 2i, and z = -2 i shown on the right. [2 pts each]
 - a) Find the matrix of the transformation that maps x to y.
 - b) Find a complex number w such that wx = y. Then draw and label w on the coordinate axes to the right.

w = _____ (in a + bi form)

c) Find a complex number v such that zx = v. Then draw and label v on the coordinate axes to the right.

v = _____ (in a + bi form)

d) Find the matrix of the transformation that maps x to v.



- 6. Consider the equation $z^4 = (z+1)^4$
 - a) Explain, using words, vector diagrams, and/or DeMoivre's Theorem, why z <u>cannot</u> be in the 1st quadrant.
 [3 pts]

b) There are 3 possible answers for z. Find all of them. Give your answers in a + bi form. [3 pts]

7. Consider the complex number z in the first quadrant, as shown in the diagram below. |z| = 2 and $Arg(z) = \theta$.



- a) Draw (z 2) onto the diagram, and draw the triangle that is created by the origin, z, and (z 2). [3 pts]
- b) Find an expression for the area of the triangle, in terms of θ . [4 pts]

8. Given the transformation matrix T below and the pre-image graphed on the right,



- a) Apply the transformation to the points and graph the image on the same axis. [3 pts]
- b) (multiple choice: circle the BEST answer) The transformation given by matrix T is: [2 pts]
 - i) a reflection, a dilation, and a shear (in that order)
 - ii) a rotation, a stretch, and a shear (in that order)
 - iii) a rotation, a dilation, and a translation (in that order)
 - iv) a reflection, a dilation, and a translation (in that order)
- 9. The maximum period of an n-post snap group is 105. Find n. Justify your answer. [3 pts]

A: Shear by 4 in y-direction
B: Shear by 4 in x-direction
C: Stretch by 4 in x-direction
D: Stretch by 4 in y-direction
E: Dilation by 4
F: Translation 4 units in x-direction

G: None of the above

i)
$$\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$$
 ii) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ iii) $\begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$ iv) $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$ v) $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ vi) $\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

11. Decide whether the set $\{1, \frac{1}{2}, 2, i, \frac{i}{2}, 2i, -1, -\frac{1}{2}, -2, -i, -\frac{i}{2}, -2i\}$ with multiplication is a group. If yes, what is the identity? If not, explain which requirement of a group is not satisfied. [2 pts]

12. Decide whether the set of all real numbers under the binary operation defined as $a \pm b = a + b - ab$ is a group. If yes, what is the identity? If not, explain which requirement of a group is not satisfied. [2 pts]