Analysis Honors – Hahn / Hlasek / Tantod
Unit 3: Polar and 3D Graphing
Big Quiz – No Calculators

61.5
65 points

When life gives you lemniscates, Michi Tanaka
Period: 6

Part I: Polar Graphing

Questions 1-2 are Multiple Choice: Circle the best answer for each problem. [2 pts each]

1. Which is a line of symmetry for $r = -5sin7\theta$?

-2 pts.

a) $\theta = \frac{2\pi}{7}$ b) $\theta = 0$ c) $\theta = \frac{\pi}{7}$ d) $\theta = \frac{15\pi}{14}$ e) $\theta = \frac{6\pi}{7}$

2. The graph of $r = \frac{1}{2}(tan\theta)sec\theta$ is a ______________

$r = \frac{1}{2}\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\right)$ $r = \frac{1}{2}\left(\frac{\sin\theta}{\cos^2\theta}\right)$ -4 pts.

$2r\cos^2\theta = \sin\theta$

a) limacon b) parabola c) ellipse

d) hyperbola e) rose curve f) none of the above

$r \cos\theta = \frac{1}{2}\frac{\sin\theta}{\cos\theta}$
$x = \frac{1}{2}\frac{\sin\theta}{\cos\theta}$
$x\cos\theta = \frac{1}{2}\sin\theta$
$x\cos\theta = \frac{y}{2r}$
$x^2 = \frac{y}{2r}$

$r = \frac{1}{2}\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos\theta}\right)$

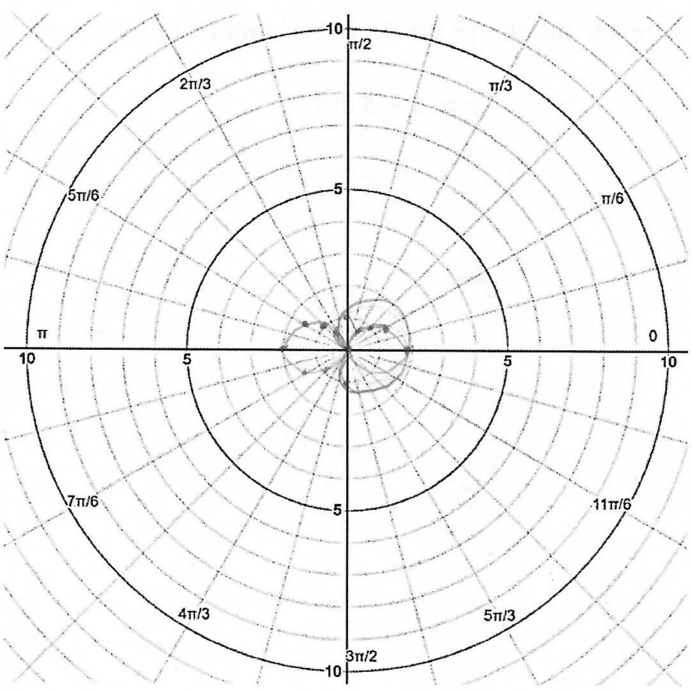
$r\cos\theta = \frac{1}{2}\left(\frac{\sin\theta}{\cos\theta}\right)$ $r\cos\theta = \frac{1}{2}\left(\frac{r\sin\theta}{r\cos\theta}\right)$

$x = \frac{1}{2}\left(\frac{y}{x}\right) = y = 2x^2$

The rest of the Polar section is Free Response. Show all your work to receive credit.

3. Find **all** the the points of intersection between the two curves: $r = 1 + cos2\theta$ and $r = 1 + cos\theta$. Give your answers as polar points. (note: $r = 1 + cos2\theta$ is NOT one of the curves that we studied this unit). [4 pts]

(the graphing space on the left is for your work, but will not be graded)



θ	r
0	2
$\frac{\pi}{4}$	$\frac{3}{2}$
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{2\pi}{3}$	$\frac{1}{2}$
$\frac{3\pi}{4}$	1

$\frac{\pi}{3}$ / $\frac{1}{2}$

(r, θ)
(0, 0)
(2, 0)

$(\frac{1}{2}, \frac{2\pi}{3})$
$(\frac{1}{2}, \frac{4\pi}{3})$

-2

$\theta = 0$
$r = 1 + \cos 2\theta$
$r = 2$

$\theta = \frac{\pi}{2}$
$r = 0$

$2\theta = \frac{\pi}{2}$
$r = 1$

$r = 1 + \cos(\frac{\pi}{4})$
$= 1 + 0$
$= 1$

$\theta = 0$
$r = 1 + \cos\theta$
$r = 2$

$\theta = \pi$
$r = 0$

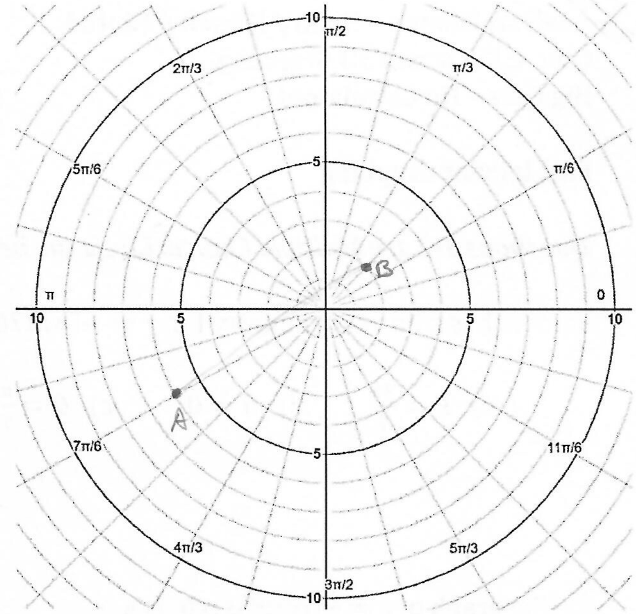
$\theta = \frac{\pi}{2}$
$r = 1$

$r = 1 + \cos(\frac{\pi}{2})$
$= 1 + 0$
$= 1$

-1 pts.

-2

4. Consider the polar points A $\left(-6,\frac{\pi}{6}\right)$ and B $\left(2,\frac{\pi}{4}\right)$.



a) Graph and label the points on the polar axis on the right. [2 pts]

b) Find the length of line segment AB. Give your answer in exact form, but no need to simplify. [2 pts]

$A: (-6\cos\frac{\pi}{6}, -6\sin\frac{\pi}{6})$ $\quad$ $B: (2\cos\frac{\pi}{4}, 2\sin\frac{\pi}{4})$

$(-3\sqrt{3}, -3)$ $\quad$ $(\sqrt{2}, \sqrt{2})$

$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(\sqrt{2}+3\sqrt{3})^2+(\sqrt{2}+3)^2} =$

Length of AB = $\sqrt{(\sqrt{2}+3\sqrt{3})^2+(\sqrt{2}+3)^2}$

4θ

5. Write the polar equation for the rose curve with eight petals, centered at the origin and passing through $(5,\frac{\pi}{8})$ with the length of each petal being $\boxed{5}$. [3 pts]

$f(\theta) = 5\sin 4\theta$

$f(\frac{\pi}{8}) = 5\sin 4(\frac{\pi}{8})$ $\quad$ $r = 5\sin 4\theta$

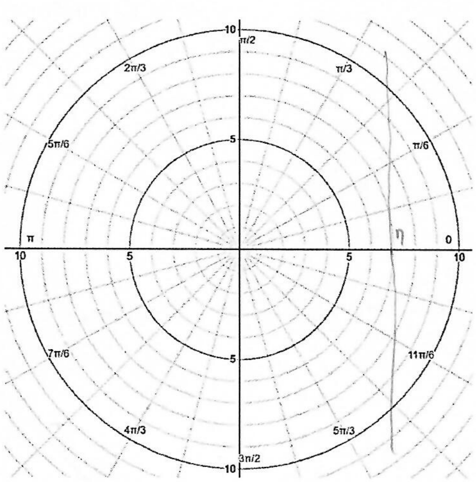
$= 5\sin\frac{\pi}{2}$

$= 5$

6. Graph each function. Then classify it according to its most specific name. [3 pts for each graph, 1 pt for name]

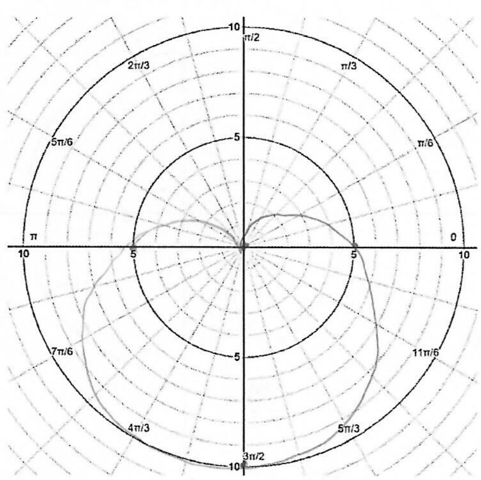
a) $r = 7\sec\theta$ $\quad$ $r = \frac{7}{\cos\theta}$ $\quad$ $r\cos\theta = 7$

$x = 7$



Name: Vertical line

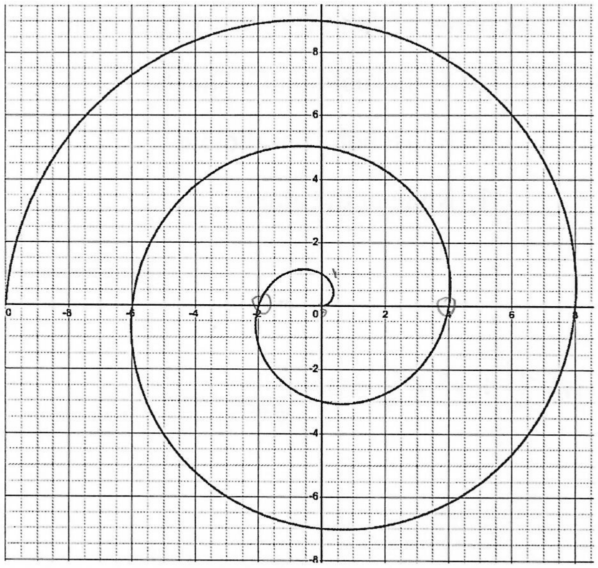
b) $r = 5 - 5\sin\theta$



Name: Cardioid Limaçon

-0

7. Write the equation of the following graph: [2 pts]



Equation: $r = \frac{2\theta}{\pi}$

$r = \frac{\theta}{\frac{\pi}{2}}$

$= \frac{2\theta}{\pi}$

polar spiral

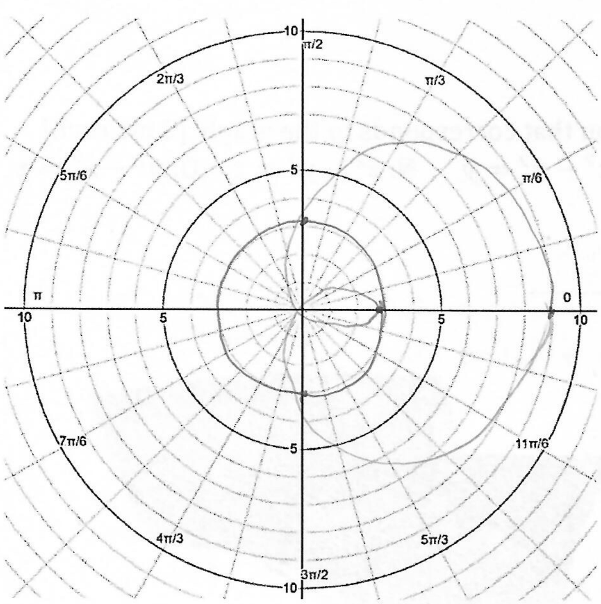
$\theta = \pi \quad r = 2$

$\theta = 2\pi \quad r = 4$

8. Find **all** the points of intersection for the system of equations: $r = 3 + 6cos\theta$ and $r = 3$. Give your answers as polar points. [3 pts]

loop

(the graphing space on the left is for your work, but will not be graded)



$3 = 3 + 6\cos\theta$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$-3 = 3 + 6\cos\theta$

$-\theta = \pi$

$\theta = -\pi = 0$

$(3, \frac{\pi}{2})$

$(3, \frac{3\pi}{2})$

$(3, 0)$

-0

Part II: 3D Graphing

9. For each equation below, write the letter that represents the best name of that 3D figure. [2 pts each]

A: Plane	**B**: Hyperboloid of 1 Sheet	**C**: Hyperboloid of 2 Sheets
D: Ellipsoid	**E**: Elliptic Cone	**F**: Hyperbolic Paraboloid
G: Elliptic Paraboloid	**H**: Parabolic Cylinder	**I**: None of the above

i) $y^2 + z^2 = x^2$ E

$y^2+z^2-x^2=0$

iv) $\frac{x^2}{4} + y^2 + \frac{z^2}{9} = 1$ D

ii) $9x^2 + 4z^2 = y$ G

$z=0$ $9x^2=y$ parabola

$x=0$ $4z^2=y$ parabola

$Y=1$ ellipse

v) $x^2 - 9y^2 = z - 8$ F

$X=0$ Parabola

$Y=0$ Parabola

$z=9$ Hyperbola

iii) $x^2 + z^2 = 7 + y^2$ B

$x^2+z^2-y^2=7$

vi) $2x + 3y + z = 1$ A

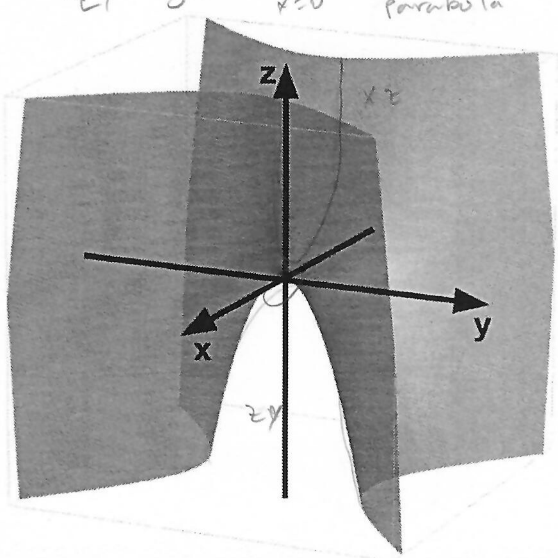
10. For each 3D graph below, write the letter with the equation that corresponds to the graph. [2 pts each]

J: $x^2 - y^2 = z$ K: $y^2 - x^2 = z$ L: $x^2 - z^2 = y$ M: $z^2 - x^2 = y$ N: $z^2 - y^2 = x$ O: $y^2 - z^2 = x$

i) J

ZY negative $Y=0$ parabola

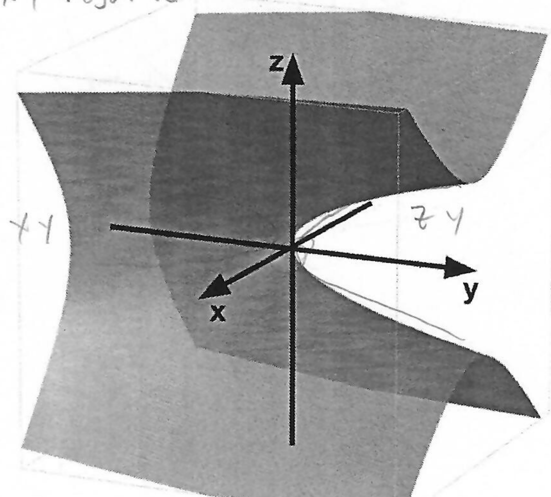
$X=0$ parabola



ii) M

XY negative $Z=0$ parabola

$X=0$ parabola



-0

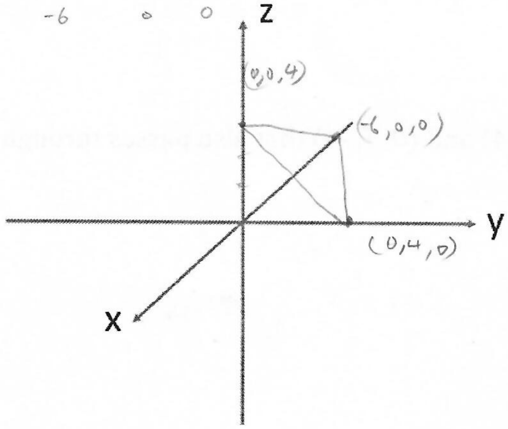
11. Sketch each 3D figure. In your sketch, <u>label at least one point</u> on the figure using its coordinates. Then state the name of each figure[3 pts each sketch, 1 pt name]

0 0 3

a) $-2x + 3y + 4z = 12$

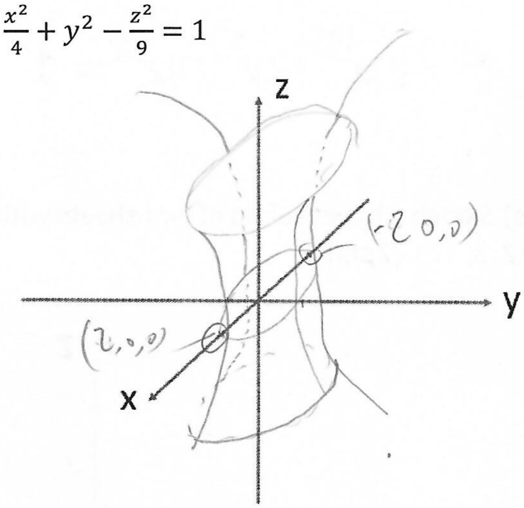
a 4 0

-6 0 0



Name: Plane

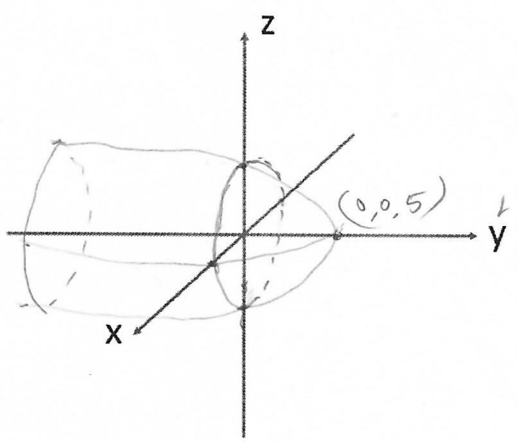
b) $\frac{x^2}{4} + y^2 - \frac{z^2}{9} = 1$



Name: Hyperboloid of 1sheet

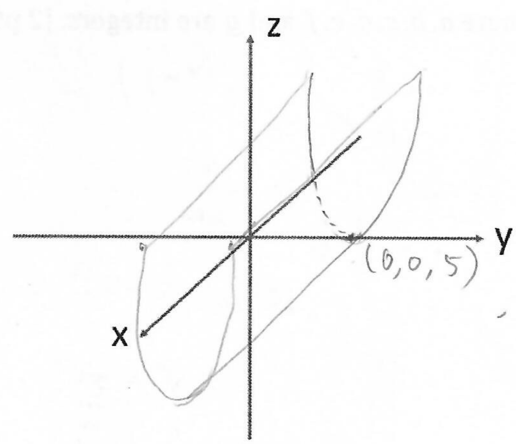
0 0 5

c) $\frac{x^2}{9} + \frac{z^2}{16} = 5 - y$



Name: Elliptic Paraboloid

d) $z = (y-5)^2$



Name: Parabolic Cylinder

/ -0

12. a) Fill in the boxes below with either "+", "-", "1", or "2" to make it the equation of a hyperboloid of two sheets with vertices on the z-axis. [1 pt]

$$\boxed{-}x^{\boxed{2}}\ \boxed{-}y^{\boxed{2}}\ \boxed{+}z^{\boxed{2}} = 1$$

$-x^2 - y^2 + z^2 = 1$

$x^2 + y^2 - z^2 = -1$ $z=0 \to$ DNE $\to$ Hyper 2 sheet.

b) Sketch a hyperboloid of two sheets with vertices (0, 1, 4) and (0, 1, -4) that also passes through the point (2, 3, 12). [2pts]

z-axis

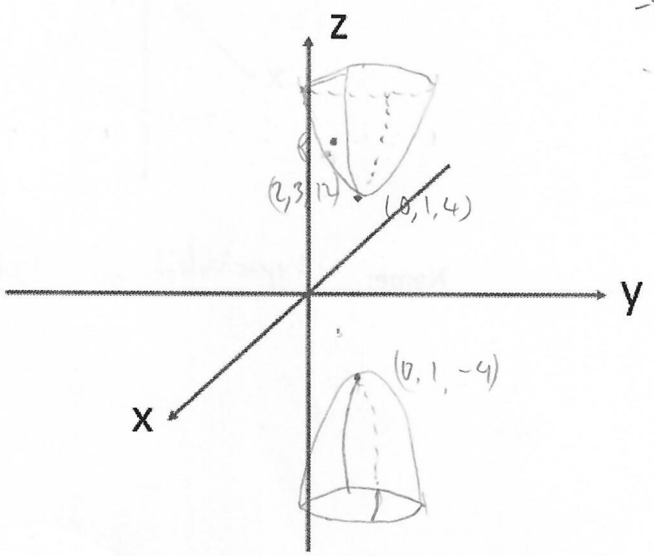
$-x^2 - y^2 + z^2 = 1$

0 1 4

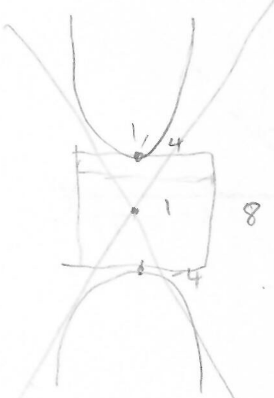
$x^2 + y^2 - z^2 = -16$

2 3 12

$4 + a - 144 = -15$



c) Write the equation of the hyperboloid from part (b) in the form $a(x+b)^2 + c(y+d)^2 + e(z+f)^2 = g$, where a, b, c, d, e, f and g are **integers**. [2 pts]



$c(y-1)$

$g=1$

$\frac{z}{64}$

8

$a(x+b)^2 + (y-1)^2 - \left(\frac{z}{8}\right)^2 = g$

$\frac{y^2}{a} - \frac{z^2}{b} = c$

$\frac{y^2}{a} - \frac{z^2}{64} = c$

$(y-1)^2 - \frac{z^2}{16} = 1$

$-2(x)^2 - 2(y-1)^2 + \frac{z^2}{16} = 1$ $-\frac{1}{2})$ not an integer

-1, $(2,3,12)$ doesn't work

Equation: $-2(x+0)^2 - 2(y-1)^2 + \frac{1}{16}(z+0)^2 = 1$

$\frac{1}{4}$ $-1\frac{1}{2}$