Analysis H – Friedland, Hlasek, Tantod
Midterm Exam 1 2024-2025
NO CALCULATORS

$\frac{59.5}{60}$

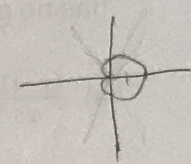
The Midterminator Neeraj G
Per 3 Date 10/8

Part 1: Polar and 3D Graphing [29 pts]

Problems 1 – 8 are multiple choice. Circle the best answer for each question [2pts each]

a:0 b:0 b:1 a:½ b:½

1. Given the points: $\left(-2, \frac{3\pi}{2}\right), \left(1, \frac{\pi}{2}\right), \left(-\frac{1}{2}, \frac{2\pi}{3}\right)$
How many of these three points in polar form represent points of intersection of $r = \cos 2\theta + 1$ and $r = 1 + \cos\theta$



a) None b) one c) two d) three

2. The curves r = 7 and $r = 11\cos 2\theta$ have ____ points of intersection

a) 2 b) 4 c) 8 d) 12 e) 16

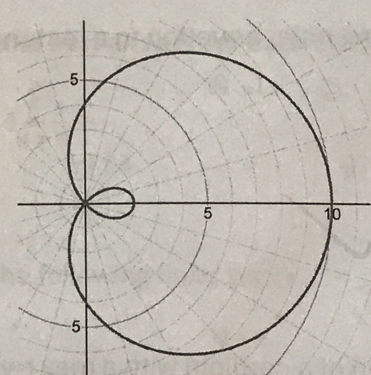
3. Polar curve with the equation $r = 6 + k\cos\theta$ (where k>0) will be a convex limacon for what values of k?

a) $3 < k < 6$ b) $k \geq 3$ c) $k \leq 3$ d) $k = 3$ e) $k < 3$

$\frac{6}{k} \geq 2$
$6 \geq 2k$
$k \leq 3$

4. Which equation best models the graph on the right?

a) $r = 4 + 6\sin\theta$
b) $r = 6 - 4\cos\theta$
c) $r = 4 - 6\sin\theta$
d) $r = 4 + 6\cos\theta$
e) $r = 6 - 4\sin\theta$



$r^2\cos^2\theta = 3r\sin\theta$

5. Convert the rectangular equation $x^2 = 3y$ to a polar equation

$r = 3\sin\theta / \cos^2\theta = 3\tan\theta\sec\theta$

a) $r = 3\tan\theta\sec\theta$ b) $r = 3\cot\theta\csc\theta$ c) $r = 3\cot\theta\sec\theta$

d) $r = 3\tan\theta\csc\theta$ e) none of these

-0

Positive

6. Given the quadric surface $16x = 4y^2 + z^2$, identify the trace for x = k, where k is a constant.

a) Circle

b) ellipse

c) hyperbola

d) parabola

e) none of the above

7. Circle the equation that best describes the following quadric surface: hyperboloid of 2 sheets that has no points for $-4 < y < 6$, but does have points for all other y-values

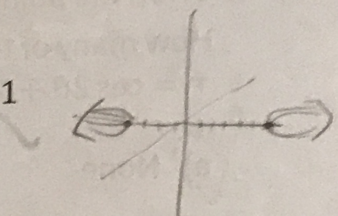
a) $\frac{(y-1)^2}{25} = \frac{x^2}{9} + \frac{z^2}{16}$

b) $\frac{x^2}{9} - \frac{z^2}{16} - \frac{(y-1)^2}{25} = 1$

c) $\frac{(y-1)^2}{25} - \frac{x^2}{9} - \frac{z^2}{16} = 1$

d) $\frac{(y-1)^2}{25} + \frac{x^2}{9} - \frac{z^2}{16} = -1$

e) $\frac{(y-1)^2}{25} + \frac{x^2}{9} + \frac{z^2}{16} = 1$



$\frac{(y-1)}{25} - \frac{x^2}{a} - \frac{z^2}{b} = 1$

8. Circle the equation that best describes the following quadric surface: hyperbolic cylinder that is parallel to the x-axis, and contains the point (2, 5, -8)

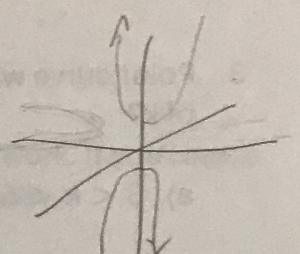
a) $\frac{z^2}{64} - \frac{y^2}{25} = 1$

b) $\frac{z^2}{32} - \frac{y^2}{25} = 1$

c) $\cancel{\frac{x^2}{4} - \frac{y^2}{25} = 1}$

d) $\cancel{\frac{x^2}{2} - \frac{y^2}{25} = 1}$

e) $\cancel{\frac{z^2}{64} + \frac{y^2}{25} = 1}$



Problems 9 -12 are free response. Please show ALL your work

9. **[2 pts]** Convert the following polar equation to a rectangular equation: $r = 10\cos\theta + 6\sin\theta$ $\cdot r$

$r^2 = 10r\cos\theta + 6r\sin\theta$

$x^2 + y^2 = 10x + 6y$

$x^2 - 10x + y^2 - 6y = 0$

$(x-5)^2 + (y-3)^2 = 34$

$\frac{25}{9}$
34

$a = b$

10. a) **[3 pts]** Write the equation of a cardioid with a max r-value of 10 and the graph has symmetry about the line $\theta = \frac{\pi}{2}$. sin

$r = 5 + 5\sin\theta$

b) **[2pts]** For your equation above, name an angle $0 \le \theta < 2\pi$ for which the curve passes through the pole (origin).

$0 = 5 + 5\sin\theta$

$\sin\theta = -1$

$\theta = \frac{3\pi}{2}$

11. **[3 pts]** Write an equation of a plane that passes through the points:
(6, 0, 0), (0, -8, 0) and (0, 0, 2)

$ax + by + cz = d$

~~$-16x + 12z$~~

$4x - 3y + 12z = 24$

~~$d = 6 \cdot (-8) \cdot 2 = -96$~~

~~$a = \frac{-96}{6} = -16$~~

~~$b = \frac{-96}{-8} = 12$~~

~~$c = \frac{-96}{2} = -48$~~

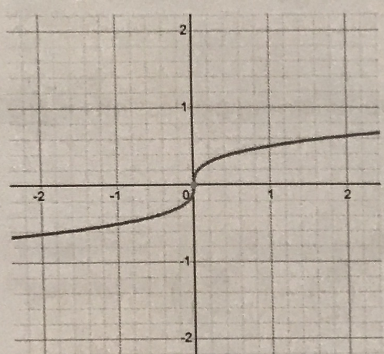
LCM: 24

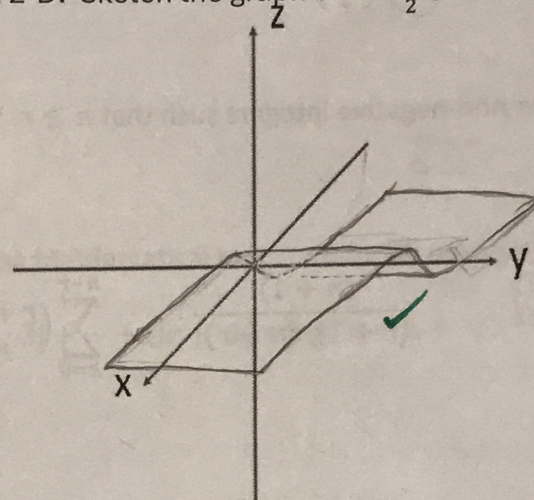
$a = \frac{24}{6} = 4$

$b = \frac{24}{-8} = -3$

$c = \frac{24}{2} = 12$

12. **[3 pts]** Below is the graph of $y = \frac{1}{2}\sqrt[3]{x}$ in 2-D. Sketch the graph of $z = \frac{1}{2}\sqrt[3]{x}$ in 3-D below.





Part 2: Algebra Through Problem Solving [31 pts]

$S = \frac{a_1 + a_n}{2} \cdot n$

Problems 13 – 21 are multiple choice. Circle the best answer for each question [2pts each]

13. Evaluate the infinite series $36 - 12 + 4 - \frac{4}{3} + \cdots$

$r = -\frac{1}{3}$

a) 27 b) 81 c) 54 d) ∞

$\frac{a_1}{1-r} = \frac{36}{1+\frac{1}{3}} = \frac{36 \cdot 3}{4}$

$= 9 \cdot 3 = 27$

14. Choose Sigma notation that matches the following finite series:
65 + 58 + 51 + 44 + ... – 110.

$a_n = 65 - 7(n-1)$

a) $\sum_{k=0}^{25}(65 - 7k)$ b) $\sum_{k=0}^{26}(72 - 7k)$ c) $\sum_{k=0}^{26}(65 - 7k)$ d) $\sum_{k=1}^{26}(65 - 7k)$

$25 \cdot 7 = 175$, $175 - 65 = 110$

$26 \cdot 7 = 182$, $182 - 65 = 117$

45
13
135
+45
585

$65 - 110 = a$

$-a = 110 - 65 = 45$

$-110 + 65 = -45$

$65 - 110 = 55$

15. Evaluate the finite series from the previous problem:
65 + 58 + 51 + 44 + ... – 110 =

$\frac{26}{2} \cdot (65 - 110)$

a) -702 b) -676 c) -585 d) -475

$S = \frac{65 - 110}{2} \cdot 26 = -45 \cdot 13$

$n = 26$

$-110 = 65 - 7(n-1)$

$-175 = -7(n-1)$

$175 / 7 = 25$

-0

16. Let $\{F_n\}$ be Fibonacci sequence. Express F_{17} in terms of F_{14} and F_{11}.

a) $F_{17} = 4F_{14} + F_{11}$ b) $F_{17} = 3F_{14} + 2F_{11}$ c) $F_{17} = 5F_{14} - F_{11}$ d) none of these

$F_{17} = F_{16} + F_{15}$

$F_{15} + F_{14} + F_{14} + F_{13}$

$2F_{14} + \underbrace{F_{15} + F_{13}}$

$\underbrace{F_{14} + F_{13} + F_{12} + F_{11}}_{F_{14}}$

$4F_{14} + F_{11}$

17. Choose the expression below that equals to

$\binom{10}{7} + \binom{9}{6} + \binom{8}{5} + \binom{7}{4} + \binom{6}{3} + \binom{5}{2} + \binom{4}{1} + \binom{3}{0}$.

$\binom{3}{0}$ $\binom{10}{7}$ $\binom{11}{7}$

a) 2^{10} b) $\binom{11}{7}$ c) F_{11} d) $\binom{11}{6}$

18. Let n and r be non-negative integers such that $n \geq r$. Which of the following expressions is **NOT** equal to $\binom{n+1}{r+1}$?

$n = 10$
$r = 0$

a) $\binom{n+1}{n-r}$ b) $\dfrac{(n+1)!}{(r+1)!\,(n-r)!}$ c) $\sum_{k=0}^{n-r} \binom{r+k}{r}$ d) $\binom{n}{r} + \binom{n}{r-1}$

$\binom{11}{1}$ $\binom{11}{10}$

n r $r+1$

$n+1$ r $r+1$ $r+2$

$\dfrac{(-4)(-5)\ldots(-20)}{17!}$ ← negative number

19. Choose an expression that is equal to $\binom{-4}{17}$.

$\dfrac{-4 \cdot -5 \ldots -16}{17!}$ negative

a) $-\binom{19}{3}$ ~~b)~~ $\binom{19}{3}$ c) $-\binom{20}{3}$ ~~d)~~ $\binom{20}{3}$

$-\dfrac{19!}{16!\;3!}$ $-\dfrac{20!}{17!\cdot 3!} = \dfrac{20 \cdot \ldots \cdot 4}{17!}$ $\dfrac{n(n-1)\ldots(n-k+1)}{k!}$

20. Which of the following binomial coefficients is **positive**?

~~a)~~ $\binom{-3}{5}$ ~~b)~~ $\binom{3}{6}$ ~~c)~~ $\binom{3}{-6}$ d) $\binom{-3}{6}$

$\dfrac{-3 \cdot -4 \cdot -5 \cdot -6 \cdot -7}{5!}$

21. Find the coefficient of x^6 in the binomial expansion of $\left(x^3 - \frac{3}{x}\right)^6$.

a) $\binom{6}{3}(-3)^3$ ~~b)~~ $\binom{18}{6}(-3)^6$ c) $\binom{6}{4}(-3)^4$ d) $\binom{6}{0}(-3)^0$

$\binom{6}{3}$

$\binom{6}{0}$ $18 - 0 = 18$

$\binom{6}{1}$ $15 - 1 = 14$

$\binom{6}{2}$ $12 - 2 = 10$

$\binom{6}{3}$ $9 - 3 = 6$

/0

Problems 22 -25 are free response. Please show ALL your work

22. **[3 pts]** Algebraically find all positive integers n that satisfy the equation
$16 \cdot (n-1)! = 5 \cdot n! + (n+1)!$ (Guess and check will be worth no points.)

$= 5 \cdot n(n-1)! + (n+1)(n)(n-1)!$

$16 = 5n + n^2 + n$

$n^2 + 6n - 16 = 0$

$(n-2)(n+8) = 0$

$n = 2, \cancel{-8}$

1/2

23. **[5 pts]** Prove the following statement using Mathematical Induction for all positive integers n. Please write neatly.

$$2 \cdot 1! + 5 \cdot 2! + 10 \cdot 3! + \cdots + (n^2+1) \cdot n! = n \cdot (n+1)!$$

Base Case: $n=1$

$(1^2+1) \cdot 1! = 1(1+1)!$

$2 = 2$ ✓

Induction Hypothesis: Assume $n=k$ is true

$2 \cdot 1! + 5 \cdot 2! + 10 \cdot 3! + \ldots + (k^2+1) \cdot k! = k(k+1)!$

Induction Step: Prove $n = k+1$ using assumption

$2 \cdot 1! + 5 \cdot 2! + 10 \cdot 3! + \ldots + (k^2+1) \cdot k! + ((k+1)^2+1) \cdot (k+1)! = (k+1)(k+2)!$

assumption

$k^2+2k+1+1$
$+2$

$k(k+1)! + ((k+1)^2+1) \cdot (k+1)!$

$(k+1)!\left[k + k^2 + 2k + 2\right] = (k+1)!\left[k^2+3k+2\right] = (k+1)!(k+1)(k+2)$

$(k+2)!$

$= (k+1)(k+2)!$ ✓

QED by induction

5/5

Problems 24-25 use The Even Numbers Triangle which is shown below.
Note that 2 is the 1st entry of the 2nd row.

1	0
2	2 4
	6 8 10
	12 14 16 18
5	20 22 24 26 28 15
	…

24. **[2 pts]** Which of the following statements is **false**?
   a) Sum of the n^{th} row is $n(n^2-1)$.
   b) The median of the n^{th} row is n^2-1.
   c) Last term in the n^{th} row is $n(n+1)-2$.
   d) First term in the n^{th} row is $n(n+1)$.

$5 \cdot 6$

25. **[3 pts]** The following holds about the Even Numbers Triangle:
- The sum of the entries in the top one row is **0**.
- The sum of the entries in the top two rows is 0 + 2 + 4 = **6**.
- The sum of the entries in the top three rows is 0 + 2 + 4 + 6 + 8 + 10 = **30**.

Find an explicit formula for the sum of the entries in the top n rows. Show all your work. If you choose to use any properties or formulas about The Odd Numbers triangle, state them clearly.

$S = n\dfrac{a_1 + a_n}{2}$ $a_1 = 0$ the last term is $n(n+1)-2$

\# of terms in top n rows: $\dfrac{n(n+1)}{2} =$

	n^2	n
n^2	n^4	n^3
n	n^3	n^2
-2	$-2n^2$	$-2n$

$$S = \frac{n(n+1)}{2} \cdot \frac{0 + n(n+1)-2}{2} = \frac{(n^2+n)(n^2+n-2)}{4}$$

$$= \frac{n^4 + 2n^3 - n^2 - 2n}{4}$$

$$\frac{2^4 + 2\cdot 2^3 - 2^2 - 2\cdot 2}{4} = \frac{16+16-4-4}{4} = \frac{24}{4} = 6$$