Hope I don't get a mid grade: __Neeraj Cummalam__

$\frac{81}{86 \text{ pts}}$ → $\frac{96.47}{100 \text{ pts}}$ Period: __3__

No Calculator Section $\frac{64}{68}$
Part 1 Probability

Multiple Choice: Circle the best answer. [2pts each]

1) Refer to the diagram on the right, which shows the number of students who take various courses. What is the probability that a randomly chosen student takes mathematics or technology, given they take science?
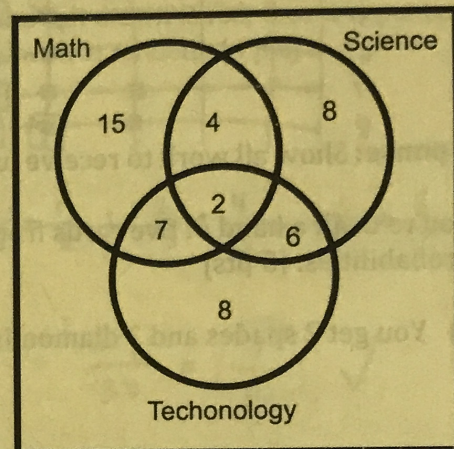
$$4 + 2 + 6 = \frac{12}{20}$$
$$4 + 2 + 6 + 8$$
$$= \frac{6}{10} = \frac{3}{5}$$

Math — 15, 4, 8 Science
7, 2, 6
8
Techonology

a) $\frac{3}{5}$ ✓

b) $\frac{3}{8}$

c) $\frac{1}{10}$

d) $\frac{4}{5}$

e) Not enough information

2) In a soccer (futbol) match, the outcomes are win, lose, or tie. Due to a nagging injury, there's a 60% chance superstar Kylian Mbappé doesn't play. If Mbappé plays, the probability of winning is 78% and the probability of losing is 10%. If he does not play, the probability of winning is 40% and the probability of a tie is 50%. Find the probability if you already know his team lost, what is the probability Mbappé played.
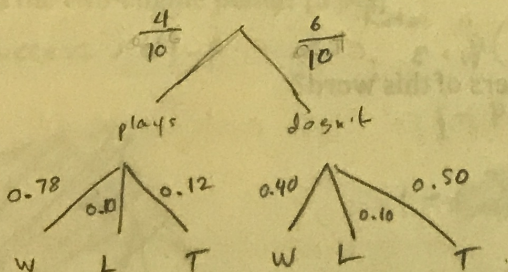
a) 2/3   b) 2/5 ✓   c) 3/5   d) 1/3   e) not enough info

$\frac{4}{10}$ — plays — 0.78 / 0.0 \ 0.12 → W L T

$\frac{6}{10}$ — dosn't — 0.40 / \ 0.10 → W L T, 0.50

$$P(\text{plays} \mid \text{lost}) = \frac{P(\text{plays} \cap \text{lost})}{P(\text{lost})}$$

$$= \frac{\frac{4}{10} \cdot \frac{1}{10}}{\frac{4}{10} \cdot \frac{1}{10} + \frac{6}{10} \cdot \frac{1}{10}} = \frac{4}{4+6} = \frac{4}{10}$$

3) Two eight-sided dice (each numbered 1 through 8) are tossed. What's the probability that the product of the rolled numbers is greater than 30 given that it is greater than 20?

a) 15/64   b) 19/28   c) 17/26   (d) 15/26   e) not enough info

$$\frac{15}{26}$$

Free response: Show all work to receive full credit. Leave your answers as expressions.

4) You're dealt a hand of five cards from a standard 52 card deck. Find the following probabilities: [5 pts]

a) You get 2 spades and 3 diamonds.

$$\frac{\binom{13}{2}\binom{13}{3}}{\binom{52}{5}}$$

b) You get 2 of any one suit and 3 of any other suit.

$$\frac{\binom{4}{1}\binom{13}{2}\binom{3}{1}\binom{13}{3}}{\binom{52}{5}}$$

c) You get two pairs, where one of the pairs is a pair of aces. (The fifth card must NOT match either pair.)

aces →   other pair   other card

$$\frac{\binom{4}{2}\binom{12}{1}\binom{4}{2}\binom{11}{1}\binom{4}{1}}{\binom{52}{5}}$$

5) Consider the word: COUNTENANCE. [4 pts]

11 total   N: 3   C: 2   E: 2

a) In how many ways can you arrange all the letters of this word?

$$\frac{11!}{3!\,2!\,2!}$$

−1

b) What is the probability that a randomly generated arrangements of the letters will have the letters "OUT" next to each other, in any order?

11−3=8+1=9

$$9 \cdot \frac{8!}{3!\,2!\,2!} \cdot \frac{1}{\frac{11!}{3!\,2!\,2!}} = \frac{9!\ 3!}{11!}$$

6) A jar contains 5 black and 8 red gumballs. You close your eyes and remove all gumballs from the jar one by one without replacement. What is the probability that the last one you choose is black?[3pts]

$$\boxed{\frac{5}{13}} \checkmark$$

7) A bag contains 1 white ball and 2 red balls. A ball is drawn at random. If the ball drawn is white, then it is put back in the bag along with another white ball. If the ball drawn is red, then it is put back in the bag along with another 2 extra red balls. A second ball is drawn at random. [4 pts]

a) Find the probability that the second ball drawn is red.
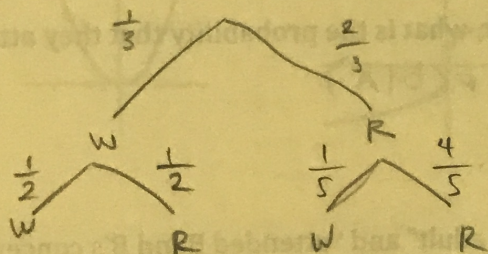
$$\frac{1}{3}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{4}{5} = \frac{1}{6} + \frac{8}{15} = \frac{5}{30} + \frac{16}{30}$$

$$= \frac{21}{30} = \boxed{\frac{7}{10}} \checkmark$$

b) If the second ball is red, what is the probability that the first ball drawn was red

$$P(1R \mid 2R) = \frac{P(1R \cap 2R)}{P(2R)} = \frac{\frac{2}{3}\cdot\frac{4}{5}}{7/10} = \frac{8}{\cancel{3}\cancel{15}}\cdot\frac{\cancel{10}^{2}}{7} = \boxed{\frac{16}{21}}$$

$$\checkmark$$

8) Doomsday Airlines books a daily service from Kansas City to St. Louis. They offer two one-way flights, one on a two-engine prop plane and another on a four-engine prop plane. Suppose that each engine fails independently of one another with some probability p. A plane will arrive safely only if at least half of its engines are in working order. For what values of p would you prefer to fly in the two-engine plane? [3 pts]

-0.5

$$P(success) = 1-p$$

$$P(2\text{ land}) > P(4\text{ land})$$

$$1 - P(0\text{ success}) > 1 - P(1\text{ success}) - P(0\text{ success})$$

$$1 - \cancel{\binom{2}{0}(1-p)^0}(p)^2 > 1 - \binom{4}{1}(1-p)^1(p)^3 - \cancel{\binom{4}{0}(1-p)^0}(p)^4$$

$$1 - p^2 > 1 - (4)(1-p)(p^3) - p^4$$

$$\left(p^2 < \frac{4(1-p)(p^3)+p^4}{p^2}\right)$$

$$1 < 4(1-p)\cdot p + p^2 \qquad \rightarrow \circled{1} < 4p - 3p^2 - 1$$

$$1 < 4p - 4p^2 + p^2 \qquad\qquad 3p^2 - 4p \cancel{\oplus} < 0$$

$$P = \frac{4 \pm \sqrt{16 - 12}}{6} = \frac{4 \pm 2}{6} = \cancel{\frac{6}{6}}, \frac{2}{6}$$

$$\boxed{P > \frac{1}{3}}$$

04 som

-0.5

9) The table below shows the concert attendance by age of a random sample of 500 people in Palo Alto. [4 pts]

	Band A	Band B	Band C	Total
Teen	60	120	120	300
Adult	28	124	48	200
Total	88	244	168	500

a) Given that a randomly selected person is an adult, what is the probability that they attended Band B's concert?

$P(B|A)$

$\dfrac{124}{200}$ ✓

b) Among the people Palo Alto, are the events "is an adult" and "attended Band B's concert" independent? Justify your answer based on probabilities calculated from the table above

A = adult
B = band B

$P(B) = \dfrac{244}{500}$

$P(B|A) = \dfrac{124}{200}$

these are not equal, so they are not independent. A affects B. ✓

10) Suppose there is a 90% chance that Mr. Friedland will be a life-long season ticket holder for the San Francisco Giants. Suppose that there is also a 5% chance he will throw out the first pitch of a Giants game one day. [5 pts]

a) If the probability of both events happening is 3%, are the two events independent? Explain clearly.

−1

0.87 0.03 0.02
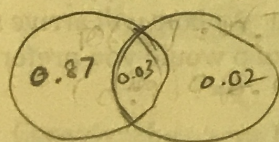
Not independent! 0.87 · 0.02 ≠ 0.03, so they affect each other.

(0.9 · 0.05 ≠ 0.03

b) What is the probability that he does not become a life-long season ticket holder and never throws out the first pitch at a Giants game?

1 − 0.87 − 0.03 − 0.02

8% ✓

c) Vin exclaims, "I bet that there's really like a 92% chance he's a life-long season ticket holder, a 10% chance he gets to throw out the first pitch and a 1% chance that both events happen." Explain clearly why these probabilities are impossible according to the laws of probability.
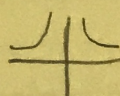
0.91 0.01 0.09
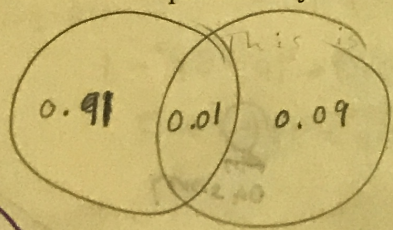
not This is not possible, because in the venn diagram, if you add everything up, the probability is greater than 1. This is not possible, because it means over 100%

−1

1) Refer to the sketches of power functions below (in the form $y = kx^p$, where k is real and p is rational) to answer the questions. To get credit for each question below you must **list all** of the graphs above which satisfy the question. Write "none" if none of the graphs satisfy the question. [1 pt each]

(1) $-\frac{3}{1}$

(2) $\frac{1}{3}$

(3) 3

(4) -2

(5) $\frac{2}{1}$

(6) $\frac{1}{3}$

(7) $\frac{1}{2}$

(8) -2 -2
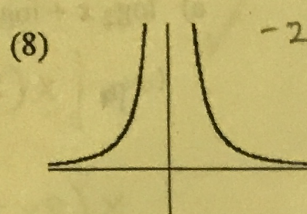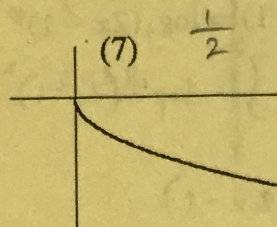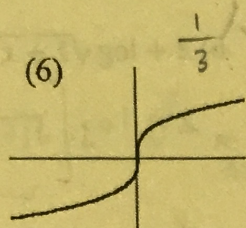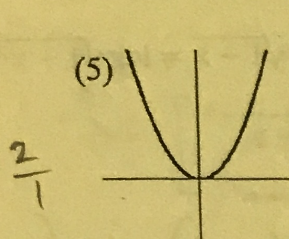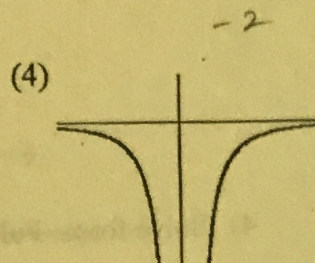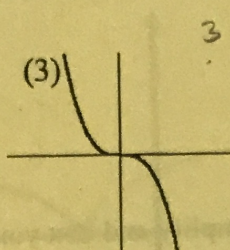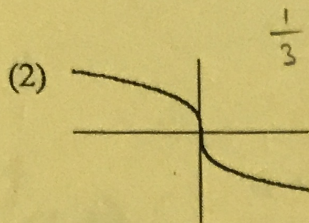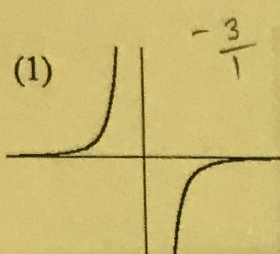
a) Which function(s) have an equation $y = kx^{-\frac{4}{3}}$?

a) $4, 8$ ✓

b) Which function(s) have asymptotes of $x = 0$ and $y = 0$?

b) $1, 4, 8$ ✓

c) In which function(s) is the value of the exponent $0 < p < 1$

c) $2, 6, 7$ ✓

d) In which function(s) is the value of the constant k negative?

d) $1, 2, 3, 4, 7$ ✓

e) Which function(s) have an equation $y = kx^p$ with $p = \frac{\text{even}}{\text{odd}} > 1$?

e) 5 ✓

f) In which function(s) is the end behavior as $x \to \infty, y \to -\infty$ and as $x \to -\infty, y \to \infty$.

f) $2, 3$ ✓

2) Given $\log_x(y) = a$ and $\log_y(z) = b$, express $\log_x(z)$ in terms of a and b. [3 pts]

$(x^a = y)^{\frac{1}{a}}$

$y^? = x$

$y^{\frac{1}{a}} = x$

$\frac{\log_y z}{\log_y x} = \frac{b}{\log_y x} = \frac{b}{1\frac{1}{a}} = \boxed{ab}$ ✓

$\frac{}{20}$

3) Determine the values of x that satisfy $\log_4(x-2) \le 2$ [2pts]

-0.5

$$x - 2 \le 16$$

$$2 < \boxed{x \le 18}$$

4) Solve for x. Fully simplify and box your final answers. [3 pts each]

a) $\log_2 x + \log_2(2x-1) = \log_4(2x-1)^2$

$\log_4\left[x(2x-1)\right]^2 = \log_4(2x-1)^2$

$x(2x-1) = (2x-1)$

$x = 1$

a) $x = \boxed{1}$ ✓

b) $2 + \log\sqrt{1+x} + 3\log\sqrt{1-x} = \log\sqrt{1-x^2}$

$2 + \log\left[\sqrt{1+x} \cdot \sqrt{1-x}^3\right] = \log(...)$

$\log 100$

$2\log\left(100 \cdot \sqrt{1+x}\sqrt{1-x} \cdot (1-x)\right) = \log\left((1+x)(1-x)\right)$

$100 \cdot \sqrt{1+x}\sqrt{1-x} \cdot (1-x) = (1+x)(1-x)$

$\left(100\sqrt{1-x} = \sqrt{1+x}\right)^2$

$10000(1-x) = 1+x$

9999

$10000 - 10000x = 1+x$

b) $x = \boxed{\dfrac{9999}{10001}}$

$x = \dfrac{}{}$

5) Solve $2e^{2x} + 5e^x - 3 = 0$. Fully simplify and box your final answer. [3pts]

$2u - 1$

$e^x = u$

$2u^2 + 5u - 3 = 0$

$\dfrac{2u^2 - u}{-3 \quad 6u \quad -3}$

$(2u-1)(u+3) = 0$

$u = e^x = \dfrac{1}{2}$ ✗

$x = \ln\left(\dfrac{1}{2}\right)$

$x = \boxed{\ln(0.5)}$ ✓
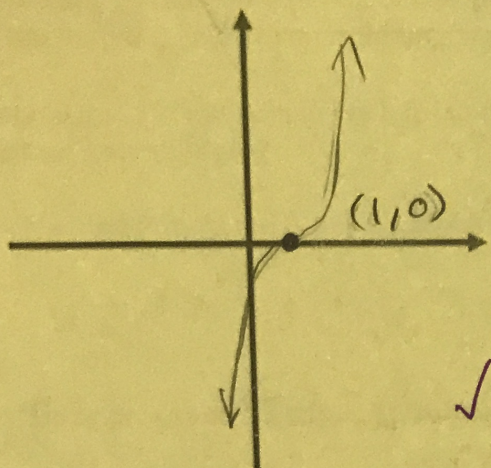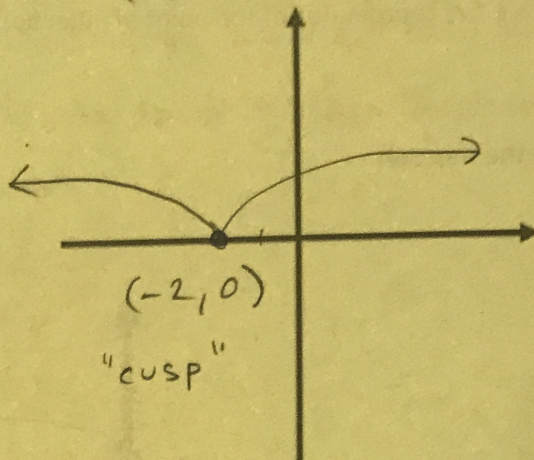
-1.5

6) Graph each power function. Your graph should be located in correct quadrants, show correct curvature, and include one point labeled with its coordinates.
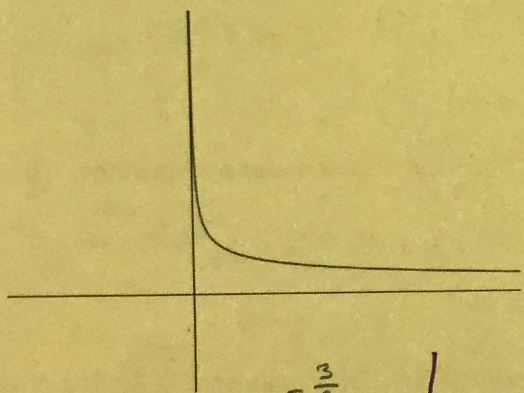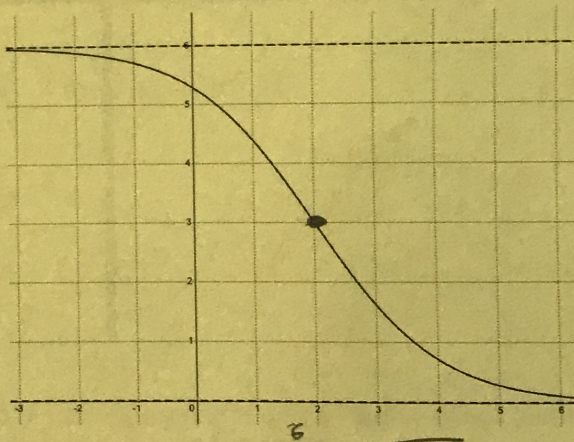[2 pts each]

a) $y = 2(x-1)^{\frac{3}{5}}$ $\frac{1}{3}$

b) $y = (-x-2)^{\frac{2}{3}}$ $(-(x+2))^{\frac{2}{3}}$

$(1,0)$ ✓

$(-2,0)$

"cusp"   ✓

7) Write an equation that will match each graph below. You may not use piecewise functions or absolute value symbols in your answer. [2 pts each]

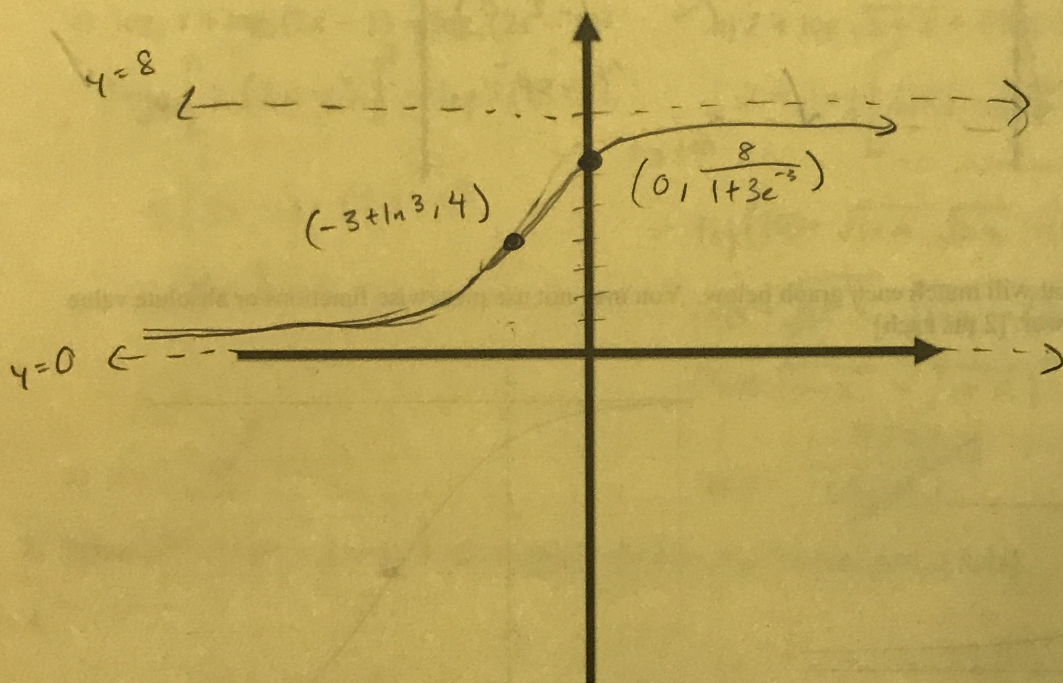Equation: $y = x^{-\frac{3}{2}}$ ✓

Equation: $y = \dfrac{6}{1+e^{(x-2)}}$ ✓

✓ 6

8) Consider the function: $y = \dfrac{8}{1+3e^{-x-3}}$. Your answers may include powers of e or natural logarithms. [1 pt per blank, 2 for the graph]

$$\dfrac{8}{1+e^{-(x+3-\ln 3)}}$$

a) What are the coordinates of the y-intercept? $\left(0, \dfrac{8}{1+3e^{-3}}\right)$ ✓

b) What are the equations of the asymptotes? $y=8$ and $y=0$ ✓

c) What are the coordinates of the point of inflection? $(-3+\ln 3, 4)$ ✓

d) Graph the function

$y=8$

$(-3+\ln 3, 4)$

$\left(0, \dfrac{8}{1+3e^{-3}}\right)$

$y=0$

✓

-2

Name: Neeraj G   Per: 3

Potentially useful formulas:

$$FV = C\left[\frac{(1+i)^n - 1}{i}\right] \qquad PV = C\left[\frac{1-(1+i)^{-n}}{i}\right]$$

For each problem, you should write the relevant formula(s), show which numbers to plug in for which variable. Then, you can use your calculator (including any probability or finance functions) to find a numerical answer

1. Assume that 13% of people are left-handed. If we select 5 people at random, find the probability of each outcome below: [8 pts]

a) The first lefty is the fifth person chosen

$$(0.87)^4 \cdot (0.13) = \boxed{0.0745} \quad \checkmark$$

b) There are **exactly** 3 lefties in the group

$$\binom{5}{3}(0.13)^3(0.87)^2 = \boxed{0.0166} \quad \checkmark$$

c) There are **some** lefties among the 5 people
$$^{1-4}$$

$n=5$

$$\text{Binom Cdf}\left(\underset{n}{5}, \underset{P}{0.13}, \underset{lower}{1}, \underset{upper}{4}\right) = \boxed{0.502} \quad \checkmark$$

d) There are **no more** than 3 lefties in the group
$$0,1,2,3$$
$$\text{Binom Cdf}\left(5, 0.13, 0, 3\right) = \boxed{0.9987} \quad \checkmark$$

2. A $525,803 mortgage paid over 30 years at 4.25% yearly interest requires monthly payment of $2586.63. Suppose you decided to make monthly payments of $3300. [6pts] $\boxed{PV}$

a) In how many months would the mortgage be completely paid off?

Finance Solver:

$I\% = 4.25$ Pmt = -3300 $\boxed{N = 235 \text{ months}} \checkmark$
PV = 525803   Ppy = 12
FV = 0

b) How much money do you save with payments of $3300 vs the payments of $2586.63?

$$2586.63 \cdot 360 - 3300 \cdot 235 = \boxed{\$155,686.80} \checkmark$$

3. Keisha stops at a coffee shop every work day (20 times a month to drink a $4.75 latte. After studying investments, she decides to skip the latte and instead invest the same amount, at the end of each month, in an annuity paying 5.7% yearly compounded monthly, if she continues this for 40 years, how much will the annuity be worth? How much of that is interest? [4 pts]

(FV)

Finance Solver :

N = 40·12

i% = 5.7

PV = 0

Pmt = 4.75 · 20

FV = ???.?1.75

Ppy = 12

$$FV = \$17,4480.98$$

interest

$17,4480.98 - 40·12·4.75^{.20} =$

$$\$172,200.98$$

interest