28/30

Analysis H — NO CALCS - 30 pts - Neeraj Gummalam probably knows what they are doing

Friedland/Hlasek/Tantod - Ver A - Per (circle): 1 2 (3) 4 5 6 7

FOR ALL PROBLEMS WRITE THE EXPRESSION THAT WOULD ANSWER EACH QUESTION. NO NEED TO SIMPLIFY ANY OF YOUR ANSWERS!

1) Consider the word: CONNECTICUT (11 letters) C: 3, N: 2, T: 2

a) (2 pts) Write the expression for the number of different arrangements of the letters if there are no restrictions.

$$\frac{11!}{3!2!2!}$$

b) (2 pts) Write the expression for the number of different arrangements if the second letter has to be a U.

U _ _ _ _ _ _ _ _ _

$$\frac{10!}{3!2!2!}$$

2) The 49ers roster has had a lot of injuries and many unsigned players want to join the team! Suppose 4 running backs, ~~3 wide receivers~~, and 6 defensive linemen are hoping to make the team, but there are limited spots available.

a) (2 pts) In how many ways can the head coach select 2 running backs, 1 wide receiver, and 3 defensive linemen?

$$\binom{4}{2}\cdot\binom{3}{1}\cdot\binom{6}{3}$$

b) (2 pts) The assistant coach wants to select the six players differently. She simply wants more defensive linemen than running backs and she is not selecting any wide receivers. In how many ways is this possible?

6 > 0
5 > 1
4 > 2

$$\binom{6}{6}\binom{4}{0}+\binom{6}{5}\binom{4}{1}+\binom{6}{4}\binom{4}{2}$$

3) (3 pts) Elliot calculates the number of combinations of a certain scenario to be 1140 while Saira discovers the number of permutations of the same scenario to be 6840. How many objects were being arranged by Saira?

$${}_nP_k = \frac{n!}{(n-k)!} = 6840$$

$${}_nC_k = \frac{n!}{k!(n-k)!} = 1140$$

$\frac{6840}{k!} = 1140$ (k chosen)

$k! = 6$ $k = 3$ objects

1140 | 6
6 | 190
0

4) (3 pts) A biologist is attempting to classify 52,000 species of insects by assigning 4 digit passcodes (any digit can be a 0 and repetition is allowed) to each species. Is it possible to classify all of the species in this manner? If so, justify. If not, how many digits for the passcodes should be used?

$10 \cdot 10 \cdot 10 \cdot 10 = 10^4 = 10000$ total

10000 < 52000

No. 4 digits gives you 10^4 (10000) unique passcodes. 10000 < 52000, so there aren't enough. The biologist should use 5 digits.

5) (2 pts) Pizza party! In how many ways can you order a 3-topping pizza if there are 8 toppings to choose from and toppings can be repeated? Order of toppings does not matter. i.e. triple pepperoni is considered acceptable (although probably unhealthy). A pizza with olives, mushrooms, and pineapple is the same as pizza with pineapple, mushrooms, and olives.

ABC = CBA

$8 \cdot 8 \cdot 8$

$$\frac{8^3}{3!}$$

-1

divide by 3! because order doesn't matter

-1

Analysis H — NO CALCS - 30 pts - Noorg G **probably knows what they are doing**

6) (2 pts) Write the expression that would find the 12th **term** of the binomial expansion: $\left(8c^3 + \frac{1}{2}b^5\right)^{100}$ if the the first term of the expansion is $(8c^3)^{100}$.

$k = 11$ $\quad k = 0$ $\quad 100 - 11 = 89$ $\quad 8 = 2^3$ $\quad 2^{267}$ $\quad 267 - 11 = 256$

$$\binom{100}{11} 8^{89} \cdot c^{267} \cdot \left(\frac{1}{2}\right)^{11} \cdot b^{55} = \binom{100}{11} 2^{256} c^{267} b^{55}$$

2^{89} $\quad 3$ $\quad 267$

7) (3 pts) Write an expression for the **coefficient** of the term that contains x^{15} for the expansion $\left(\frac{2}{x^2} - x^3\right)^{10}$.

k	a	b	b − a
0	20	0	−20
1	18	3	−15
2			
7	6	21	15
8	4	24	20
9	2	27	25
10	0	30	30

$$-\binom{10}{7} \cdot 2^3$$

8) The WNBA All-Star rosters are being constructed. Let's call them Team A and Team B. Each roster will contain 8 players. There's 30 outstanding players trying out for those 16 total spots.

a) (2 pts) Are there $\binom{30}{16}$ ways of setting up the two teams? Explain why or why not.

No. $\binom{30}{16}$ just gives the # of combinations for 16 players in a "pool" of both teams. It doesn't give you the # of ways you can split those 16 people into two teams of 8.

b) (2 pts) Suppose we know that Angel Reese and Caitlin Clark will definitely be chosen but CANNOT be on the same team with one another. (It's a whole thing.) In how many ways can the two rosters be chosen?

28 players
7 per team

$$\binom{28}{7}\binom{21}{7} \times 2$$

divide by two because it can give A or B

c) (1 pt) Team A forms a circle (called a huddle) before the game starts. In how many different ways could Team A huddle?

$\frac{8!}{8}$ ← because each can be offset by 1, 8 times $= 7!$

9) (4 pts) Suppose you're rolling an eight sided die with numbers from 1 to 8 and a six sided die with numbers from 1 to 6 simultaneously. Find the probability of:

a) Rolling a **sum** greater than 10

$\frac{10}{48} = \frac{5}{24}$

b) Rolling a **product** divisible by 3

$\frac{24}{48} = \frac{1}{2}$

	1	2	3	4	5	6	7	8
1	✗	✗	b	4	5	b	7	8
2	✗	4	b			b		
3	b	b	b	b	b	b	b	b
4			b			b	a	
5			b			a	a	
6	b	b	b	b	a	a	a	