Analysis H 2024-2025
Friedland / Hlasek / Tantod
Unit 3 (Probability) Quiz 2
NO CALCULATORS

31
32 pts

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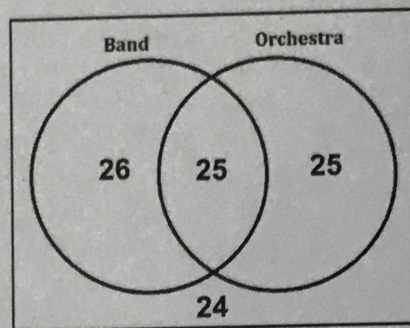
is *expected* to do well on this quiz
Date: 10/31/24 Period: 3

For Problems 1 and 2: The Venn diagram given on the right shows the responses of 100 students who were surveyed about whether they play in the band or the orchestra at their school. B represents band and O represents orchestra.



1. **[3 pts]** Which of the following probabilities are equal to $\frac{1}{2}$? Circle **all** the apply.

~~b)~~ $P(B)$ ~~b)~~ $P(B')$ ~~c)~~ $P(O \mid B)$

d) $P(O)$ e) $P(O')$ f) $P(B \mid O)$ ✓

2. Express the following probabilities as fractions. No need to reduce.

d) **[1 pt]** $P(B' \cap O) = \frac{25}{100}$ ✓

e) **[1 pt]** $P(B' \cup O') = \frac{75}{100}$ ✓
$(B \cap O)'$

f) **[2 pts]** $P(B \mid O') = \frac{26}{50}$ ✓

3. **[3 pts]** I draw a hand of 4 cards from a deck of sixteen cards that consists of **only** Jacks, Queens, Kings, and Aces. What is the probability that I have exactly three Queens, given that I have at least one Ace? Leave your answer as an expression that may include factorials, powers, and/or binomial coefficients.

-1

$P(3Q \mid 1+A) = \frac{P(3Q \cap 1+A)}{P(1+A)}$

$\frac{\binom{4}{3}\binom{4}{1}}{\binom{16}{4}-\binom{12}{4}}$

$\frac{\binom{4}{3}}{\binom{\cancel{15}\,4}{3}}$

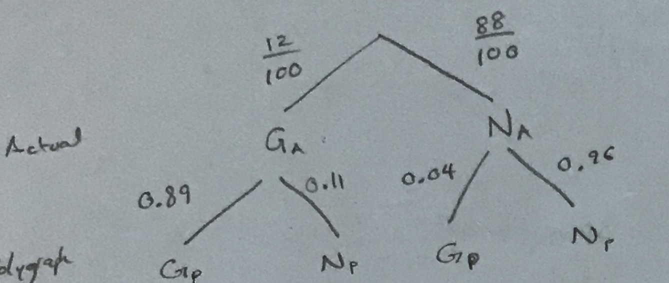
15 cards left because you have at least one ace - not exactly one ace.

-1

4. **[3 pts]** During a power blackout, 100 people are arrested on suspicion of rioting. Each person is given a polygraph test. From past experience, it is known that the polygraph is 89% reliable when administered to a guilty suspect and 96% reliable when given to someone who is innocent. Suppose that of the 100 persons taken into custody, only 12 were actually involved in any wrongdoing (guilty). **Write the algebraic expression** that represents the probability that a suspect is innocent given that the polygraph says he is guilty.

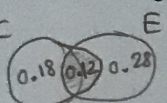
$$P(N_A | G_P) = \frac{P(N \cap G_P)}{P(G_P)} \quad \frac{(0.88)(0.04)}{(0.12)(0.89)+(0.88)(0.04)}$$

✓



$$\hookrightarrow = \frac{P(N_A)\cdot P(G_P | N_A)}{P(G_A)\cdot P(G_P|G_A) + P(N_A)\cdot P(G_P|N_A)}$$

5. Spike is struggling with his academics at the moment. His chances of passing chemistry are 0.3, his chances of passing English are 0.4 and his chances of passing both are 0.12.



a) **[2 pts]** Are the events "Spike passes chemistry" and "Spike passes English" independent? **In one sentence,** explain your answer after showing all relevant calculations.

Yes, they are independent, meaning passing one class doesn't affect his probability of passing the other.

$P(C)\cdot P(E) = P(C \cap E)$ $\quad (0.3)(0.4) = 0.12$ ✓ ✓

b) **[2 pts]** Are the events "Spike passes chemistry" and "Spike passes English" mutually exclusive? **In one sentence,** justify your answer mathematically.

No, mutually exclusive means that the chance of both happening $P(C \cap E) = 0$, but the problem states $P(C \cap E) = 0.12$ meaning both events can happen (not mutually exclusive). ✓

c) **[2 pts]** What is the probability that he **fails** both subjects?

$P(F_c) = 0.7$

$P(F_E) = 0.6$

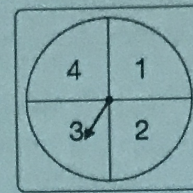
$P(F_c \cap F_E) = P(F_c)\cdot P(F_E) = (0.7)(0.6) = 0.42$ ✓

$1 - P(C) - P(E) + P(C \cap E) = 1 - 0.3 - 0.4 + 0.12 = 0.42$

✓

-0

6. **[5 pts]** The diagram on the right shows a fair spinner such that each number from 1 to 4 is equally likely to be spun. The spinner is spun **12** times.
For each statement below, write if it is **TRUE** or **FALSE**.
If a statement is false, change something about it to make it true.



I highlighted my chages oh, thanks

a) The probability that the number 3 is spun exactly once is $\binom{12}{1}\cdot\frac{1}{4}\cdot\left(\frac{3}{4}\right)^{11}$ True ✓

b) The probability that the number 3 is spun exactly 6 times is $\cancel{\frac{1}{2}}$ False ✓

$\binom{12}{6}\left(\frac{1}{2}\right)^6\left(\frac{3}{4}\right)^6$ ✓

c) The probability that the number 3 is not spun at all is $\left(\frac{3}{4}\right)^{12}$ True ✓

$\cancel{\binom{12}{12}}\left(\frac{3}{4}\right)^{12}\cancel{\left(\frac{1}{4}\right)^0}$

d) The probability that the number 3 is spun exactly twice is not the same as the probability that the number 3 is spun exactly ten times. False ✓

$\cancel{\binom{12}{2}}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^{10}$ $\cancel{\binom{12}{10}}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^2$ ✓

$\left(\frac{3}{4}\right)^8 \neq \left(\frac{1}{4}\right)^8$

e) The probability that the number 3 is spun less than 10 times is

$$1-\underbrace{\binom{12}{11}\cdot\left(\frac{1}{4}\right)^{11}\cdot\frac{3}{4}}_{P_{11}}-\underbrace{\binom{12}{12}\cdot\left(\frac{1}{4}\right)^{12}}_{P_{12}}-\binom{12}{10}\cdot\left(\frac{1}{4}\right)^{10}\cdot\left(\frac{3}{4}\right)^2$$ False ✓

$1-P(12)-P(11)-P(10)$ ✓

$1-\binom{12}{12}\left(\frac{1}{4}\right)^{12}-\binom{12}{11}\left(\frac{1}{4}\right)^{11}\left(\frac{3}{4}\right)-\binom{12}{10}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^2$

7. **[3 pts]** A fair six-sided die is rolled n times, where n is a positive integer. The probability that a four was rolled exactly 9 times is the same as the probability that a four was rolled exactly 8 times. Find the value of n. Your answer should be a single integer. Please show your work in a neat and organized fashion.

$\binom{n}{9}\left(\frac{1}{6}\right)^9\left(\frac{5}{6}\right)^{n-9}=\binom{n}{8}\left(\frac{1}{6}\right)^8\left(\frac{5}{6}\right)^{n-8}$

$(n-8)-(n-9)$

$=\cancel{n}-8\cancel{-n}+9=1$

$\binom{n}{9}\left(\frac{1}{6}\right)\left(\frac{5}{6}\right)^{n-9}=\binom{n}{8}\left(\frac{5}{6}\right)^{n-8}$

$6\cdot\left[\binom{n}{9}\left(\frac{1}{6}\right)=\binom{n}{8}\left(\frac{5}{6}\right)\right]$

$\cancel{(n!)}\cancel{(8!)}(n-8)!=5\cancel{(n!)}(9\cancel{8!})(n-9)!$

$(n-8)!=5\cdot9\cdot(n-9)!$

$(n-8)\cancel{(n-9)!}=5\cdot9\cancel{(n-9)!}$

$5\cdot9\cancel{8!}(n-9)!=\cancel{8!}(n-8)!$

$\binom{n}{9}=5\binom{n}{8}$

$5\cdot9=n-8$

$\frac{n!}{9!(n-9)!}=\frac{5(n!)}{8!(n-8)!}$

$45=n-8$

$n-8=45$

$n=53$

$n=45+8$

$n=53$ ✓

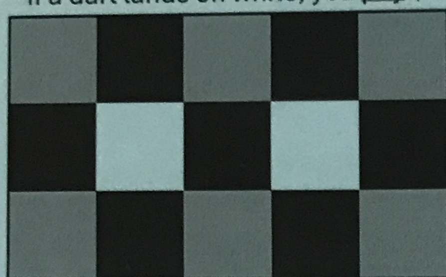
-0

8. **[5 pts]** Your friend Dave gives you a choice of playing one of the following games. Which game would you play? Explain your answer and show all relevant calculations.

Game 1:

There is an equally likely chance that a falling dart will land anywhere on the dartboard below that consists of 15 squares.

- If a dart lands on black, you gain $30.
- If a dart lands on gray, you gain $20.
- If a dart lands on white, you pay $15.

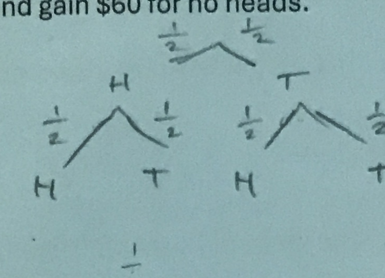


total 15
2 white
7 black
6 gray

Game 2:

You flip a coin twice and record the number of heads that occur.

You gain $40 for 2 heads, pay $5 for at least 1 head, and gain $60 for no heads.



Your work related to **Game 1:**

	P	$	Product
black	$\frac{7}{15}$	30	$2 \cdot 7 = 14$
gray	$\frac{6}{15}$	20	$\frac{6}{3} \cdot 4 = 8$
white	$\frac{2}{15}$	−15	−2

E = $20 ✓

Your work related to **Game 2:**

	P		product
2 heads	$\frac{1}{4}$	$40-5=35$	$\frac{35}{4}$
1 head	$\frac{1}{4}+\frac{1}{4}=\frac{1}{2}$	−5	$\frac{-5}{2} = \frac{-10}{4}$
0 heads	$\frac{1}{4}$	60	$+\frac{60}{4}$

$\frac{85}{4}$

↳ E = $21.25 ✓

Conclusion:

I would play the second game, because the expected value is higher (Game 2 has $21.25, Game 1 has only $20).

↳ I will earn more. ✓

−0