

Analysis Honors 24/25 NO CALCULATORS
 Friedland / Hlasek / Tantod
 Unit 2 (AtPS) Quiz

27 28
 30 points

Triangulator Nearay G
 Period 3

Odd Number Triangle

(Reminder: In the Odd Number Triangle, the row with [3 5] is the 2nd row.)

1. Write "true" or "false" for each statement. (1 pt each)

- a) The top n rows of the Odd Number Triangle include exactly $\frac{n(n-1)}{2}$ terms.
 b) The sum of all the terms in row n of the Odd Number Triangle is n^3 .
 c) The last term in row n of the Odd Number Triangle is $n^2 + n - 1$.

False ✓ 3 5
True ✓ 7 9
True ✓ 11

2. Which row of the Odd Number Triangle includes the number 195? Explain your answer. (2 pts)

First term: $n^2 + n + 1$ plugging in 14: first: $196 + 14 + 1 = 183$
 last term: $n^2 + n - 1$ last: $196 + 14 - 1 = 209$ 195 is in this range

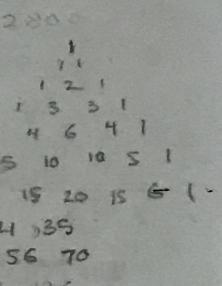
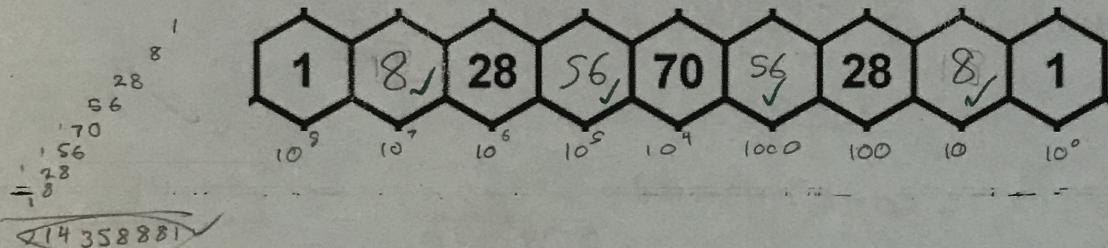
I knew $14^2 = 196$ so it made sense that 195 would be in this range

row 14 ✓

Pascal's Triangle

(Reminder: In Pascal's Triangle, the row with [1 2 1] is the 2nd row.)

3. Complete all four blanks in the following row of Pascal's Triangle given that $11^8 = 214358881$. (2 pts)



4. Simplify the expression below into a single term. (2 pts)

$$\binom{18}{2} + \binom{18}{4} + \binom{18}{6} + \binom{18}{8} + \cdots + \binom{18}{16} + \binom{18}{18} = 2^{17} \checkmark$$

5. Simplify the expression below into a single binomial coefficient. (2 pts)

$$\frac{\binom{9}{6} \cdot \binom{10}{5} \cdot \binom{11}{7}}{\binom{9}{5} \cdot \binom{10}{7}} = \binom{11}{6} \checkmark$$

Fibonacci Numbers

(Reminder: $F_0 = 0, F_5 = 5$)

$$F_{a+b+1} = F_{a+1} F_{b+1} + F_a F_b$$

$$F_{2a+1} = F_{a+1}^2 + F_a^2$$

6. Circle all the expressions below that are equal to F_{17} . (2 pts)

a) $F_0 + F_2 + F_4 + \cdots + F_{16} + 1$ ✓
 F_1 F_3 $F_{17} + 1$

b) $F_{18} - F_1$

c) $(F_8)^2 + (F_9)^2$

d) $1F_{11} + 3F_{12} + 3F_{13} + 1F_{14}$

$$\underbrace{F_{11} + F_{12} + F_{13}}_{15} + \underbrace{F_{12} + F_{13} + F_{14}}_{16} + \underbrace{F_{13} + F_{14}}_{15}$$

7. Find a positive integer m that satisfies the following identity. Show all your work. (3 pts)

$$\binom{4}{0} \cdot F_{21} + \binom{4}{1} \cdot F_{22} + \binom{4}{2} \cdot F_{23} + \binom{4}{3} \cdot F_{24} + \binom{4}{4} \cdot F_{25} = F_m.$$

$$\begin{aligned} & F_{21} + 4F_{22} + 6F_{23} + 4F_{24} + F_{25} = F_{21} + F_{22} + F_{23} + F_{24} + F_{25} \\ & = \underbrace{F_{23} + F_{24}}_{F_{25}} + \underbrace{F_{21} + F_{22} + F_{23} + F_{24}}_{F_{25}} + F_{25} \\ & = F_{25} + F_{26} + F_{27} + F_{28} \\ & = F_{27} + F_{28} + F_{29} \\ & = F_{28} + F_{29} \\ & = F_{29} \\ & m = 29 \end{aligned}$$

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Sequences and Series

8. Consider geometric sequence $\{a_n\}$ such that $a_3 = 500$ and $a_6 = -32$.

a) Find the common ratio r of this geometric sequence. Give an exact completely simplified answer. (3 pts)

$$\frac{a_6 = a_1 \cdot r^5 = -32}{(a_3 = a_1 \cdot r^2 = 500)} = r^3 = -\frac{32}{500} < \frac{-16}{250} = \frac{-8}{125}$$

$$a_3 = a_1 \cdot \left(-\sqrt[3]{\frac{8}{125}}\right)^2 = a_1 \cdot \left(-\frac{8}{125}\right)^{\frac{2}{3}}$$

$$a_1 = \frac{500}{\left(\frac{8}{125}\right)^{\frac{2}{3}}}$$

$$r = -\sqrt[3]{\frac{8}{125}}$$

b) Find the sum $a_1 + a_2 + \dots + a_9 + a_{10}$. Leave your answer as a numerical expression without sigma notation. (2 pts)

$$S_n = a_1 \cdot \frac{(1-r^n)}{1-r} = \frac{500}{\left(\frac{8}{125}\right)^{\frac{2}{3}}} \cdot \left(\frac{1 - \left(\frac{8}{125}\right)^{\frac{10}{3}}}{1 + \sqrt[3]{\frac{8}{125}}} \right) \checkmark$$

9. Find a positive integer N that satisfies the following identity. Show algebraic work to receive full credit. (3 pts)

$$\sum_{k=1}^N (-3+2k) = 168$$

$$S_n = \frac{-1 - 3 + 2n}{2} \cdot n = 168$$

$$\frac{168}{4} = \frac{32}{2} = 16$$

$$\begin{array}{c} \text{A triangle with base } n \text{ and height } 2n. \\ \text{The formula for the area of a triangle is } \frac{1}{2} \times \text{base} \times \text{height}. \\ \text{So, } \frac{1}{2} \times n \times 2n = n^2. \end{array}$$

$$- \frac{4 + 2n}{2} \cdot n = 168$$

$$\frac{52}{2} = 14$$

$$(n-2)(n) = 168$$

$$n^2 - 2n - 168 = 0$$

$$n = \frac{2 \pm \sqrt{4 + 4 \cdot 168}}{2}$$

$$\frac{2 \pm 26}{2} = \frac{28}{2} = 14$$

The Fun Problem! ☺

10. In the triangle below, number 8 is the 1st term on the 2nd row.

$$\begin{array}{ccccccc} 5 & & & 5 & & & \\ 8 & 11 & & 19 & & & \\ 14 & 17 & 20 & & 51 & & \\ 23 & 26 & 29 & 32 & & & \\ 35 & 38 & 41 & 44 & 47 & & \\ 50 & 53 & 56 & 59 & 62 & 65 & \\ 68 & 71 & 74 & 77 & 80 & 83 & 86 \end{array}$$

$$\frac{103}{4} = 25.75$$

a) Find the last term on the 7th row. (1pt)

$$\frac{3}{2}(7^2) + \frac{3}{2} \cdot 7 + 2 =$$

$$86$$

$$\frac{32}{5} = 6.4$$

b) Find the sum of the 8th row. Show all your work. Using brute force (listing all the terms and adding them up) will receive no credit. Give your answer as a single number. (2 pts)

$$S = \frac{(\text{first} + \text{last}) \cdot n}{2} \cdot n \checkmark$$

$$5, 8, 11, 14, 17, 20, 23$$

$$t = an^2 + bn + c$$

$$S = an^2 + bn + c$$

$$8 = 4a + 2b + c$$

$$14 = 9a + 3b + c$$

$$20 = 16a + 4b + c$$

$$26 = 25a + 5b + c$$

$$32 = 36a + 6b + c$$

$$38 = 49a + 7b + c$$

$$44 = 64a + 8b + c$$

$$47 = 81a + 9b + c$$

$$50 = 100a + 10b + c$$

$$53 = 121a + 11b + c$$

$$56 = 144a + 12b + c$$

$$59 = 169a + 13b + c$$

$$62 = 196a + 14b + c$$

$$65 = 225a + 15b + c$$

$$68 = 256a + 16b + c$$

$$71 = 289a + 17b + c$$

$$74 = 324a + 18b + c$$

$$77 = 361a + 19b + c$$

$$80 = 396a + 20b + c$$

$$83 = 433a + 21b + c$$

$$86 = 475a + 22b + c$$

$$86 = 516a + 23b + c$$

$$86 = 557a + 24b + c$$

$$86 = 600a + 25b + c$$

$$86 = 643a + 26b + c$$

$$86 = 686a + 27b + c$$

$$86 = 739a + 28b + c$$

$$86 = 792a + 29b + c$$

$$86 = 845a + 30b + c$$

$$86 = 908a + 31b + c$$

$$86 = 971a + 32b + c$$

$$86 = 1034a + 33b + c$$

$$86 = 1107a + 34b + c$$

$$86 = 1180a + 35b + c$$

$$86 = 1253a + 36b + c$$

$$86 = 1326a + 37b + c$$

$$86 = 1400a + 38b + c$$

$$86 = 1473a + 39b + c$$

$$86 = 1546a + 40b + c$$

$$86 = 1620a + 41b + c$$

$$86 = 1693a + 42b + c$$

$$86 = 1766a + 43b + c$$

$$86 = 1840a + 44b + c$$

$$86 = 1913a + 45b + c$$

$$86 = 1986a + 46b + c$$

$$86 = 2060a + 47b + c$$

$$86 = 2133a + 48b + c$$

$$86 = 2206a + 49b + c$$

$$86 = 2279a + 50b + c$$

$$86 = 2353a + 51b + c$$

$$86 = 2426a + 52b + c$$

$$86 = 2500a + 53b + c$$

$$86 = 2573a + 54b + c$$

$$86 = 2646a + 55b + c$$

$$86 = 2720a + 56b + c$$

$$86 = 2793a + 57b + c$$

$$86 = 2866a + 58b + c$$

$$86 = 2940a + 59b + c$$

$$86 = 3013a + 60b + c$$

$$86 = 3086a + 61b + c$$

$$86 = 3160a + 62b + c$$

$$86 = 3233a + 63b + c$$

$$86 = 3306a + 64b + c$$

$$86 = 3379a + 65b + c$$

$$86 = 3453a + 66b + c$$

$$86 = 3526a + 67b + c$$

$$86 = 3600a + 68b + c$$

$$86 = 3673a + 69b + c$$

$$86 = 3746a + 70b + c$$

$$86 = 3820a + 71b + c$$

$$86 = 3893a + 72b + c$$

$$86 = 3966a + 73b + c$$

$$86 = 4040a + 74b + c$$

$$86 = 4113a + 75b + c$$

$$86 = 4186a + 76b + c$$

$$86 = 4260a + 77b + c$$

$$86 = 4333a + 78b + c$$

$$86 = 4406a + 79b + c$$

$$86 = 4480a + 80b + c$$

$$86 = 4553a + 81b + c$$

$$86 = 4626a + 82b + c$$

$$86 = 4700a + 83b + c$$

$$86 = 4773a + 84b + c$$

$$86 = 4846a + 85b + c$$

$$86 = 4920a + 86b + c$$

$$86 = 5000a + 87b + c$$

$$86 = 5073a + 88b + c$$

$$86 = 5146a + 89b + c$$

$$86 = 5220a + 90b + c$$

$$86 = 5293a + 91b + c$$

$$86 = 5366a + 92b + c$$

$$86 = 5440a + 93b + c$$

$$86 = 5513a + 94b + c$$

$$86 = 5586a + 95b + c$$

$$86 = 5660a + 96b + c$$

$$86 = 5733a + 97b + c$$

$$86 = 5806a + 98b + c$$

$$86 = 5880a + 99b + c$$

$$86 = 5953a + 100b + c$$

$$86 = 6026a + 101b + c$$

$$86 = 6100a + 102b + c$$

$$86 = 6173a + 103b + c$$

$$86 = 6246a + 104b + c$$

$$86 = 6320a + 105b + c$$

$$86 = 6393a + 106b + c$$

$$86 = 6466a + 107b + c$$

$$86 = 6540a + 108b + c$$

$$86 = 6613a + 109b + c$$

$$86 = 6686a + 110b + c$$

$$86 = 6760a + 111b + c$$

$$86 = 6833a + 112b + c$$

$$86 = 6906a + 113b + c$$

$$86 = 6980a + 114b + c$$

$$86 = 7053a + 115b + c$$

$$86 = 7126a + 116b + c$$

$$86 = 7200a + 117b + c$$

$$86 = 7273a + 118b + c$$

$$86 = 7346a + 119b + c$$

$$86 = 7420a + 120b + c$$

$$86 = 7493a + 121b + c$$

$$86 = 7566a + 122b + c$$

$$86 = 7640a + 123b + c$$

$$86 = 7713a + 124b + c$$

$$86 = 7786a + 125b + c$$

$$86 = 7860a + 126b + c$$

$$86 = 7933a + 127b + c$$

$$86 = 8006a + 128b + c$$

$$86 = 8080a + 129b + c$$

$$86 = 8153a + 130b + c$$

$$86 = 8226a + 131b + c$$

$$86 = 8300a + 132b + c$$

$$86 = 8373a + 133b + c$$

$$86 = 8446a + 134b + c$$

$$86 = 8520a + 135b + c$$

$$86 = 8593a + 136b + c$$

$$86 = 8666a + 137b + c$$

$$86 = 8740a + 138b + c$$

$$86 = 8813a + 139b + c$$

$$86 = 8886a + 140b + c$$

$$86 = 8960a + 141b + c$$

$$86 = 9033a + 142b + c$$

$$86 = 9106a + 143b + c$$

$$86 = 9180a + 144b + c$$

$$86 = 9253a + 145b + c$$

$$86 = 9326a + 146b + c$$

$$86 = 9400a + 147b + c$$

$$86 = 9473a + 148b + c$$

$$86 = 9546a + 149b + c$$

$$86 = 9620a + 150b + c$$

$$86 = 9693a + 151b + c$$

$$86 = 9766a + 152b + c$$

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