Analysis Honors 24/25 NO CALCULATORS
Friedland / Hlasek/ Tantod
Unit 2 (AtPS) Quiz

~~27~~ 28
30 points

Triangulator Neeraj G
Period 3

Odd Number Triangle **(Reminder: In the Odd Number Triangle, the row with [3 5] is the 2nd row.)**

1. Write "true" or "false" for each statement. (1 pt each)

a) The top n rows of the Odd Number Triangle include exactly $\frac{n(n-1)}{2}$ terms. False ✓

b) The sum of all the terms in row n of the Odd Number Triangle is n^3. True ✓

c) The last term in row n of the Odd Number Triangle is $n^2 + n - 1$. True ✓

1
3 5
7 9 11

2. Which row of the Odd Number Triangle includes the number 195? Explain your answer. (2 pts)

First term: $n^2 - n + 1$
Last term: $n^2 + n - 1$

plugging in 14: first: $196 - 14 + 1 = 183$
last: $196 + 14 - 1 = 209$ } 195 is in this range

I knew $14^2 = 196$ so it made sense that 195 would be in this range

row 14 ✓

Pascal's Triangle **(Reminder: In Pascal's Triangle, the row with [1 2 1] is the 2nd row.)**

3. Complete all four blanks in the following row of Pascal's Triangle given that $11^8 = 214358881$. (2 pts)

1	8 ✓	28	56 ✓	70	56 ✓	28	8 ✓	1
10^8	10^7	10^6	10^5	10^4	1000	100	10	10^0

1
8
28
56
70
56
28
8
214358881

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35
1 8 28 56 70

4. Simplify the expression below into a single term. (2 pts)

$$\binom{18}{2} + \binom{18}{4} + \binom{18}{6} + \binom{18}{8} + \cdots + \binom{18}{16} + \binom{18}{18} = 2^{17}$$ ✓

5. Simplify the expression below into a single binomial coefficient. (2 pts)

$$\frac{\binom{9}{6}\cdot\binom{10}{5}\cdot\binom{11}{7}}{\binom{9}{5}\cdot\binom{10}{7}} = \binom{11}{6}$$ ✓

$F_{a+b+1} = F_{a+1}F_{b+1} + F_aF_b$

$F_{2a+1} = F_{a+1}^2 + F_a^2$ ✓

Fibonacci Numbers **(Reminder: $F_0 = 0$, $F_5 = 5$)**

6. **Circle all the expressions below that are equal to F_{17}.** (2 pts)

a) $\cancel{F_0} + F_2 + F_4 + \cdots + F_{16} + 1$ ~~b)~~ $F_{18} - F_1$ c) $(F_8)^2 + (F_9)^2$ d) $1F_{11} + 3F_{12} + 3F_{13} + 1F_{14}$

$\underbrace{F_1}_{F_3} \to F_{17} + 1$

$F_{11} + F_{12} + F_{12} + F_{12} + F_{13} + F_{13} + F_{13} + F_{14}$
13 14 14 15
15 16

7. Find a positive integer m that satisfies the following identity. Show all your work. (3pts)

$$\binom{4}{0}\cdot F_{21} + \binom{4}{1}\cdot F_{22} + \binom{4}{2}\cdot F_{23} + \binom{4}{3}\cdot F_{24} + \binom{4}{4}\cdot F_{25} = F_m.$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1

$F_{21} + 4F_{22} + 6F_{23} + 4F_{24} + F_{25} = \underbrace{F_{21} + F_{22}}_{F_{23}} + F_{22} + F_{22} + F_{22} + F_{23} + F_{23} + F_{23} + F_{23} + F_{23} + F_{23} + F_{23} + F_{24} + F_{24} + F_{24} + F_{24} + F_{25}$

$= F_{23} + F_{24} + F_{24} + F_{24} + F_{25} + F_{25} + F_{25} + F_{26}$

$= F_{25}$ F_{26} F_{27} $\to F_{25} + F_{26} + F_{26} + F_{27}$

F_{26} F_{27} F_{28} F_{29}

F_{24} F_{24} F_{25} F_{25} F_{26}

$\boxed{m = 29}$ ✓

/0

Sequences and Series

8. Consider geometric sequence $\{a_n\}$ such that $a_3 = 500$ and $a_6 = -32$.

a) Find the common ratio r of this geometric sequence. Give an exact completely simplified answer. (3 pts)

$a_6 = a_1 \cdot r^5 = -32$ $\quad = r^3 = -\frac{32}{500} = \frac{-16}{250} = \frac{-8}{125}$

$(a_3 = a_1 \cdot r^2 = 500)$ $\quad -1$

$a_3 = a_1 \cdot \left(-\sqrt[3]{\frac{8}{125}}\right)^2 = a_1 \cdot \left(-\frac{8}{125}\right)^{\frac{2}{3}}$ $\quad a_1 = \frac{500}{\left(-\frac{8}{125}\right)^{(2/3)}}$

$\boxed{r = -\sqrt[3]{\frac{8}{125}}}$

b) Find the sum $a_1 + a_2 + \cdots + a_9 + a_{10}$. Leave your answer as a numerical expression without sigma notation. (2 pts)

$S_n = a_1 \cdot \left(\frac{1-r^n}{1-r}\right) = \frac{500}{\left(-\frac{8}{125}\right)^{(2/3)}} \cdot \left(\frac{1-\left(\frac{8}{125}\right)^{\frac{10}{3}}}{1+\sqrt[3]{\frac{8}{125}}}\right)$ ✓

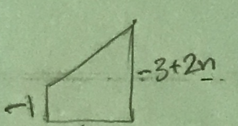
$n = 10$

9. Find a positive integer N that satisfies the following identity. Show algebraic work to receive full credit. (3 pts)

$$\sum_{k=1}^{N}(-3+2k) = 168$$

$S_n \quad \frac{-1-3+2n}{2} \cdot n = 168$

$\frac{-4+2n}{2} \cdot n$



$(n-2)(n) = 168$

$n^2 - 2n - 168 = 0 \quad n = \frac{2 \pm \sqrt{4+4 \cdot 168}}{2} \quad \frac{2+26}{2} = \frac{28}{2} = 14$

$168 \times 4 = 672$; $26 \times 26 = 676$ ✓

$\boxed{N = 14}$

The Fun Problem! ☺

10. In the triangle below, number 8 is the 1st term on the 2nd row.

5 $\quad$ 5
8 11 $\quad$ 19
14 17 20 $\quad$ 51
23 26 29 32
35 38 41 44 47
50 53 56 59 62 65 ✓
68 71 74 77 80 83 86

$103 \times 4 = 412$

a) Find the last term on the 7th row. (1pt)

$\frac{3}{2}(7^2) + \frac{3}{2} \cdot 7 + 2 =$ $\quad 86$

$32 \times 3 = 96$

b) Find the sum of the 8th row. Show all your work. Using brute force (listing all the terms and adding them up) will receive no credit. Give your answer as a single number. (2 pts)

$S = \frac{\text{first} + \text{last}}{2} \cdot n$ ✓ $\quad -1 \quad$ so, its $\frac{3}{2}n^2 - \frac{3}{2}n + 5$

5, 8, 14, 23 $\quad t = an^2 + bn + c$

3, 6, 9; 3, 3

$5 = a + b + c$
$8 = 4a + 2b + c$
$14 = 9a + 3b + c$

$3 = 3a + b \rightarrow b = 3 - 3a = -\frac{3}{2}$
$6 = 5a + b = 2a + 3 \rightarrow a = \frac{3}{2} \quad b = 5$

$S_8 = \frac{\frac{3}{2} \cdot 8^2 - \frac{3}{2} \cdot 8 + 5 + \frac{3}{2} \cdot 8^2 + \frac{3}{2} \cdot 8 + 2}{2} \cdot 8 = \frac{3 \cdot 8^2 + 7}{2} \cdot 8 = (96+7)4 = 412$ ✓

c) Find the explicit formula for the last term in n^{th} row in terms of n. Show all your work. (3 pts)

5, 11, 20, 32, 47
6, 9, 12, 15
3, 3, 3

$5 = a + b + c$
$11 = 4a + 2b + c$
$20 = 9a + 3b + c$

$6 = 3a + b \rightarrow b = 6 - 3a$
$9 = 5a + b = 5a + 6 - 3a = 2a + 6$
$a = \frac{3}{2}$
$b = \frac{3}{2}$
$c = 2$ ✓

$= an^2 + bn + c$

$\frac{3}{2}n^2 + \frac{3}{2}n + 2$