

Please show ALL your work and simplify ALL your answers

1. Convert the rectangular points into polar coordinates using a positive r and $0 \leq \theta < 2\pi$.

(2 pts each)

a) $(-\sqrt{15}, \sqrt{5})$

$$r = \sqrt{x^2 + y^2} = \sqrt{15 + 5} = \sqrt{20} = 2\sqrt{5}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{5}}{-\sqrt{15}}\right)$$

$$\frac{-\sqrt{5}}{\sqrt{15}} = -\sqrt{\frac{1}{3}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3} \Rightarrow \theta = \frac{5\pi}{6}$$

$(2\sqrt{5}, \frac{5\pi}{6})$ ✓

b) $(-10b, -10b)$ where b is a positive constant

$$r = \sqrt{x^2 + y^2} = \sqrt{200b^2} = 10b\sqrt{2}$$

$$\theta = \frac{5\pi}{4}$$

$(10b\sqrt{2}, \frac{5\pi}{4})$ ✓

2. Convert the polar coordinates into rectangular coordinates. (2 pts each)

a) $(-5, \frac{2\pi}{3})$

$$(-5 \cos \frac{2\pi}{3}, -5 \sin \frac{2\pi}{3})$$

$$(-5 \cdot -\frac{1}{2}, -5 \cdot \frac{\sqrt{3}}{2})$$

$(\frac{5}{2}, -\frac{5\sqrt{3}}{2})$ ✓

b) $(20, \frac{3\pi}{2})$

$$(20 \cos \frac{3\pi}{2}, 20 \sin \frac{3\pi}{2})$$

$$(20 \cdot 0, 20 \cdot -1)$$

$(0, -20)$ ✓

3. Convert the polar equation into a rectangular equation. (2 pts each)

a) $r = -5 \sec \theta$

$$r = -5 \cdot \frac{1}{\cos \theta}$$

$$r \cos \theta = -5$$

$x = -5$ ✓

b) $r^2 \cos 2\theta = 1$

$$r^2 (2\cos^2 \theta - 1) = 1$$

$$2r^2 \cos^2 \theta - r^2 = 1$$

$$2(r \cos \theta)^2 = 1 + r^2$$

$$2x^2 = 1 + x^2 + y^2$$

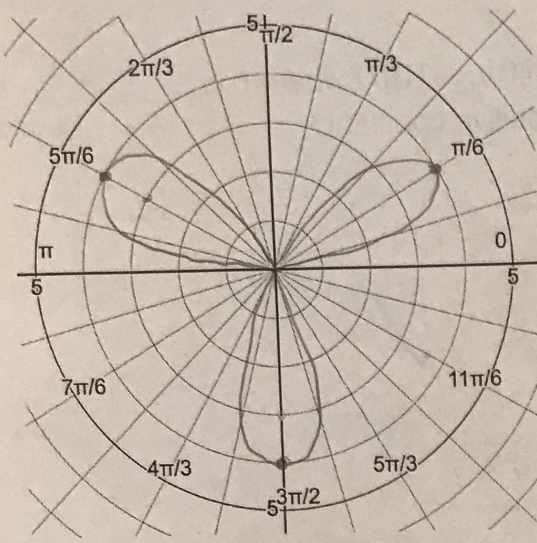
$x^2 - y^2 = 1$ ✓

*no need to simplify to a particular form

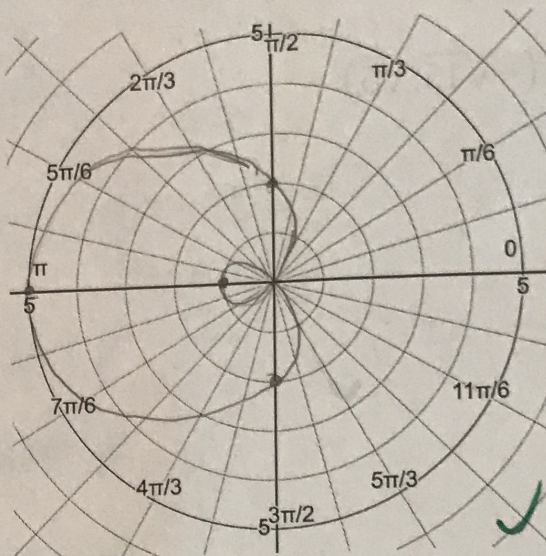
0

Directions: Graph each equation. Then classify each according to its most specific name.
(2 pts for each graph, 1 pt for each name)

4. $r = 4 \sin 3\theta$



5. $r = 2 - 3 \cos \theta$



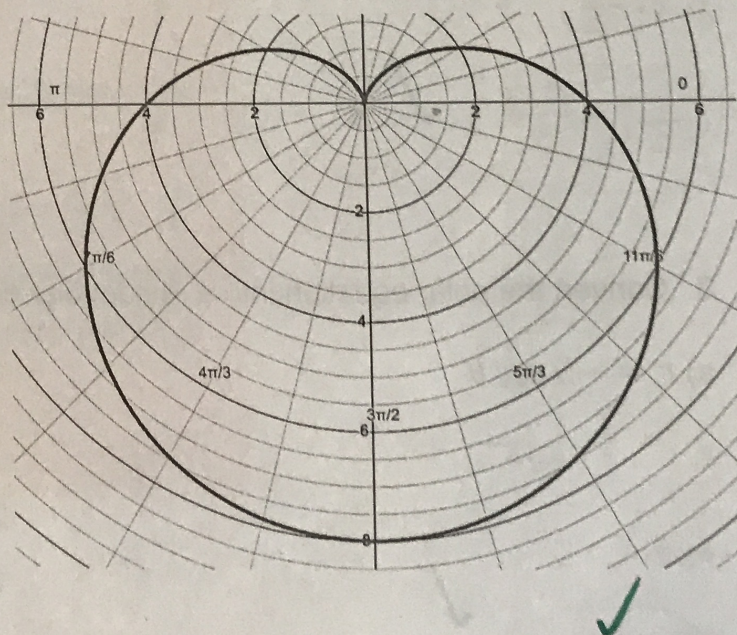
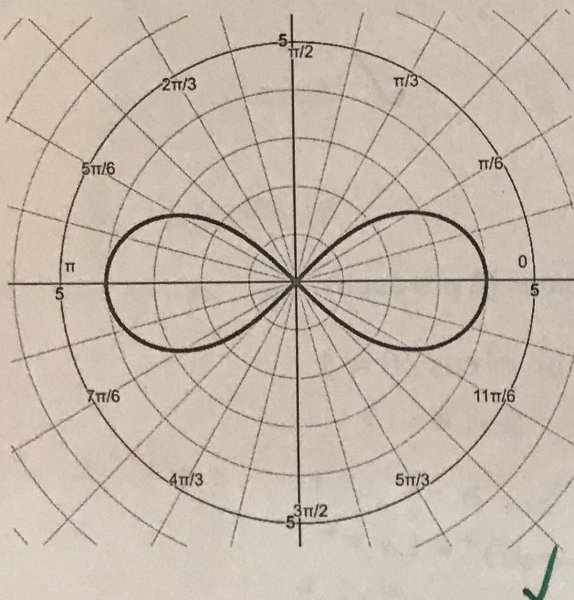
Name that curve: sine rose

Name that curve: cosine limaçon w/ inner loop

(You will receive no credit for names such as "Sheila" or "Joey"... ☺)

5. Write the equation of each graph provided. Then classify each graph according to its most specific name. (2 pts for equation, 1 for name)

a)



Equation: $r^2 = 16 \cos 2\theta$

Equation: $r = 4 - 4 \sin \theta$

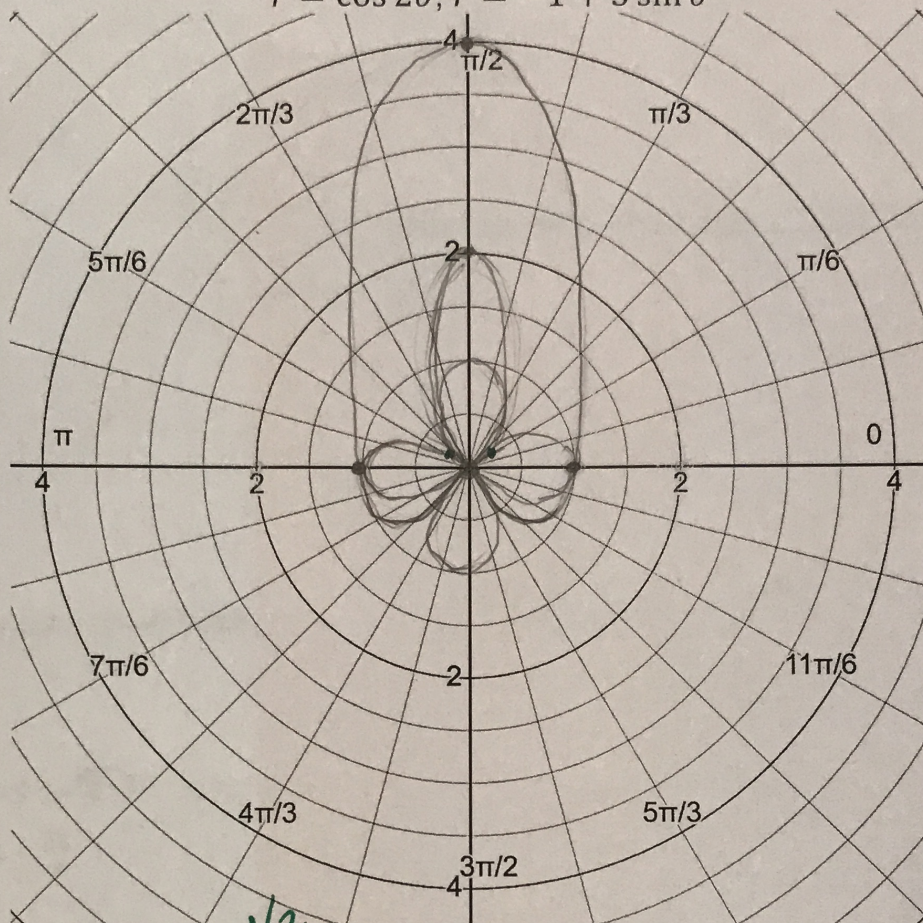
Name: cosine lemniscate

Name: sine cardioid

6. Find all the points of intersection for the pair of curves with the following polar equations.

Show all work graphically and algebraically. (6pts)

$$r = \cos 2\theta, r = -1 + 3 \sin \theta$$



$$\cos 2\theta = -1 + 3 \sin \theta$$

$$1 - 2 \sin^2 \theta = -1 + 3 \sin \theta$$

$$2 \sin^2 \theta - 3 \sin \theta + 2 = 0$$

$$u = \sin \theta$$

$$2u^2 - 3u - 2 = 0$$

	$2u^2$	u
-2	$-4u$	-2

$$(2u+1)(u-2) = 0$$

$$u = \sin \theta = -\frac{1}{2}$$

$$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$\cos\left(2 \cdot \frac{7\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(2 \cdot \frac{11\pi}{6}\right) = -\frac{1}{2}$$

Where did the $\frac{\pi}{3}$ and $\frac{5\pi}{3}$ come from?

$(-\frac{1}{2}, \frac{5\pi}{3})$ becomes this

All the points of intersection are:

$$(0,0) \quad (1,0) \quad (1,\pi) \quad \left(\frac{1}{2}, \frac{\pi}{6}\right) \quad \left(-\frac{1}{2}, \frac{5\pi}{6}\right)$$

follow through??

-1.5