KEY

Analysis "Fake Final Exam" 2020

1. The derivative of an exponential growth curve such as $y=5^x$ most closely resembles (obviously no Desmos just use your noggin)

a) another exponential growth curve b) an exponential decay curve c) a log curve
d) The reflection of an exponential growth curve over the x axis
e) The reflection of an exponential decay curve over the x axis.

2. Consider three functions f(x), g(x) and h(x) where $h(x)=g(\sqrt{f(x)})$

If f(1) = 9, f'(1) = -2, g'(3) = 4, g'(9)=5, g(1) = 10 calculate h'(1).

$h' = g'(\sqrt{f(x)}) \cdot \frac{1}{2}(f(x))^{-1/2} \cdot f'(x)$

$h'(1) = g'(\sqrt{f(1)}) \cdot \frac{1}{2}(f(1))^{-1/2} \cdot f'(1)$

$= g'(3) \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot -2$

$= 4 \cdot -1/3 = \boxed{-4/3}$

3. If g(x) is a second degree polynomial where g(0) = 5, g'(2)=3 and g''(15) = -6 write g(x) in standard form.

$g \to x^2$
$g' \to x$
$g'' \to$ constant

$g''(15) = -6$

$g' = -6x + C;\ g'(2) = -6(2) + C = 3$

$g' = -6x + 15$

$g = -3x^2 + 15x + C;\ g(0) = 0 + 0 + C = 5$

$\boxed{g = -3x^2 + 15x + 5}$

4. Find $\frac{d^{121}y}{dx^{121}}$ (the 121st derivative) if y = sin(2x)

odd #; 120 is divisible by 4 ∴ look at 1st derivative model, w/ 2^{121}

a) $-2\sin(2x)$ b) $-2^{121}\cos(2x)$ $\boxed{\text{c) } 2^{121}\cos(2x)}$ d) $2^{121}\sin(2x)$ e) $-2^{121}\sin(2x)$

$y' = 2\cos 2x$ $y'' = -2^2 \sin 2x$ $y''' = -2^3 \cos 2x$ $y'''' = 2^4 \sin 2x$ repeat

5. Given the piecewise linear graph of $f(x)$ below, find $h'(1)$ if $h(x)=f(3x^2)$. Assume the function continues on infinitely in both directions.

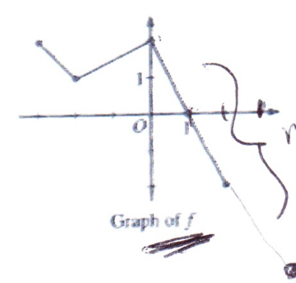
$h' = f'(3x^2) \cdot 6x$

a) 12 $\boxed{\text{b) -12}}$ c) 6 d) -6 e) -3

$h'(1) = f'(3) \cdot 6$

$= -2 \cdot 6$

$= -12$



Graph of f

need slope: -2

6. **Wind chill** (w in degrees Fahrenheit), is defined as the temperature a person feels when the velocity of the wind (v, in mph) is factored in. On a blustery 32 degree Fahrenheit day, the wind chill can be given by: $w(v) = 55.6 - 22.1v^{0.16}$

$\frac{dw}{dV} = (-22.1)(0.16)V^{0.16-1} = -3.536\,V^{-.84}$

a) Find the value of v at which the instantaneous rate of change of w is equal to the average rate of change of w over the interval v: [5, 60].

$\frac{dw}{dV} = \frac{f(60)-f(5)}{60-5}$

$-3.536\,V^{-.84} = \frac{55.6 - 22.1(60)^{.16} - [55.6 - 22.1(5)^{.16}]}{55}$

use calculator

$-3.536\,V^{-.84} = -0.253797$

$V^{-.84} = 0.071775$

$\ln V = 3.13597$

$e^{3.13597} = V$

$\boxed{V = 23.011 \text{ mph}}$

b) At time t = 0, the wind velocity is 20 mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time ($\frac{dw}{dt}$) at t = 3 hours? Include units. Show work clearly.

$t = 0 \quad V(0) = 20$

Dimensional Analysis

we want $\frac{dw}{dt} = \frac{dw}{dV} \cdot \frac{dV}{dt}$

we know $\frac{dw}{dV}$ expression

we can get $\frac{dV}{dt}$

$V(0) = 20$

$\frac{dV}{dt} = 5$

$V = 5t + C$; since $V(0) = 20$ then $C = 20$

$V = 5t + 20$

$V(3) = 35$

$\frac{dw}{dt} = \frac{dw}{dV} \cdot \frac{dV}{dt}$

$\frac{dw}{dt} = -3.536V^{-.84} \cdot 5$

$\left.\frac{dw}{dt}\right|_{t=3} = -3.536(35)^{-.84} \cdot 5$

$= \boxed{-0.8922 \frac{\text{degrees}}{\text{hr}}}$

7. Suppose that $g'(x) = \frac{3}{2\sqrt{x}} - (3x-2)^4 + 5\sin(9x)$, with $g(0)=1$. Find:

$g' = \frac{3}{2}x^{-1/2} - (3x-2)^4 + 5\sin(9x)$

a) $g''(x) = \boxed{-\frac{3}{4}x^{-3/2} - 12(3x-2)^3 + 45\cos(9x)}$

b) $g(x) = 3x^{1/2} - \frac{1}{15}(3x-2)^5 - \frac{5}{9}\cos(9x) + C$

$= \boxed{3x^{1/2} - \frac{1}{15}(3x-2)^5 - \frac{5}{9}\cos(9x) - \frac{26}{45}}$

$g(0) = 1$

$0 - \frac{1}{15}(-2)^5 - \frac{5}{9} \cdot 1 + C = 1$

$\frac{32}{15} - \frac{5}{9} + C = 1$

$C = -\frac{26}{45}$

8. Knowing that $\lim_{x\to 2}\frac{x^3 - ax^2 + bx - 2}{x-2} = -1$ solve for a, b. [7] Show all work please.

we know that the difference quotient should $= \frac{0}{0}$ $\therefore$ $x^3 - ax^2 + bx - 2 = 0$ when $x = 2$

we know that the algebraically simplified expression $= -1$ when $x = 2$

$$\begin{array}{r|rrrr} 2 & 1 & -a & b & -2 \\ & & 2 & 4-2a & 8-4a+2b \\ \hline & 1 & 2-a & 4-2a+b & 0 \leftarrow \text{known} \end{array}$$

$\frac{\cancel{(x-2)}(x^2 + x(2-a) + 4-2a+b)}{\cancel{(x-2)}} = -1$ when $x = 2$

$\begin{cases} 8 - 4a + 2b - 2 = 0 \\ 4 + 2(2-a) + 4 - 2a + b = \end{cases}$

Solve system to get

$\boxed{\begin{array}{l} a = 5 \\ b = 7 \end{array}}$

9. Consider a function that satisfies the following: At x = 4, the value of the function is 1, and the slope of the function is 1.

a) Let $f(x) = ax^2$, where a is a nonzero constant. Show that it is not possible to find a value for a so that f meets the requirement above. [3]

$f(x) = ax^2 = 1$ when $x = 4 \therefore a = \frac{1}{16}$

$f'(x) = 2ax = 1$ when $x = 4 \therefore a = \frac{1}{8}$

$\frac{1}{16} \neq \frac{1}{8}$

OR $f(x) = ax^2 = 1$ when $x = 4$ $\cancel{\rightarrow f(x) = \frac{1}{16}x^2}$

$f(x) = \frac{1}{16}x^2$

$f'(x) = \frac{1}{8}x$ when $x = 4$ $f'(4) = \frac{1}{2}$

$\frac{1}{2} \neq$ Given 1

b) Let $h(x) = \frac{x^n}{k}$ where k is a nonzero constant and n is a positive integer. Find the values of k and n so that h meets the requirement above. [5]

$h(4) = \frac{4^n}{k} \overset{\text{Given}}{=} 1 \longrightarrow k = 4^n$

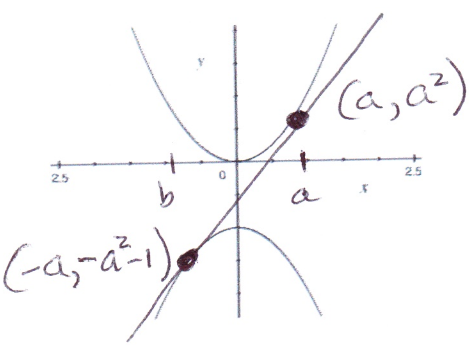
$\boxed{k = 4^4 = 256}$

$h'(x) = \frac{n}{k} \cdot x^{n-1}$

$h'(4) = \frac{n}{k} \cdot 4^{n-1} \overset{\text{Given}}{=} 1 \rightarrow \frac{n}{k} \cdot \frac{4^n}{4} = 1 \rightarrow \frac{n}{4^n} \cdot \frac{4^n}{4} = 1 \rightarrow \frac{n}{4} = 1 \Rightarrow n = 4$

10. Find the equation of the line with positive slope that is tangent to both f(x) and g(x) below.

$f(x) = x^2$, and $g(x) = -x^2 - 1$



$f'(a) = g'(b)$

$2(a) = -2(b)$

$\therefore a = -b$

write 2 expressions of slope of tangent line, and equate them.

$$\frac{f(a) - f(-a)}{a - -a} = f'(a)$$

$$\frac{a^2 - (-a^2 - 1)}{2a} = 2a$$

$$\frac{2a^2 + 1}{2a} = 2a$$

$$2a^2 + 1 = 4a^2$$

$$2a^2 = 1$$

$a = \frac{1}{\sqrt{2}}$ point: $\left(\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

Slope: $f'(a) = 2 \cdot \frac{1}{\sqrt{2}}$

line: $\boxed{y - \frac{1}{2} = \frac{2}{\sqrt{2}}\left(x - \frac{1}{\sqrt{2}}\right)}$