Analysis
Calculus – Chapter 3
Chain Rule and Trig Functions

Name______________________

Per__________Date______________

Find the derivative of the following:

1. $y=\left(x^3-\dfrac{7}{x}\right)^{-2}$

$\dfrac{dy}{dx}=-2\left(x^3-\frac{7}{x}\right)^{-3}\cdot\left(3x^2+7x^{-2}\right)$

2. $f(x)=\sqrt{4+3\sqrt{x}}$

$f'(x)=\frac{1}{2}\left(4+3\sqrt{x}\right)^{-\frac{1}{2}}\cdot\left(\frac{3}{2}x^{-\frac{1}{2}}\right)$

3. $g(x)=\cos^2\left(3\sqrt{x}\right)$

$g'(x)=2\cos\left(3\sqrt{x}\right)\cdot\left(-\sin\left(3\sqrt{x}\right)\right)\cdot\left(\frac{3}{2}x^{-\frac{1}{2}}\right)$

4. $h(x)=\sqrt{3x-\sin^2(4x)}$

$h'(x)=\frac{1}{2}\left(3x-\sin^2(4x)\right)^{-\frac{1}{2}}\cdot\left(3-2\sin(4x)\cdot(\cos(4x))\cdot(4)\right)$

5. $f(x)=\left[x^4-\cos\left(4x^2-2\right)\right]^{-4}$

$f'(x)=-4\left[x^4-\cos\left(4x^2-2\right)\right]^{-5}\left[4x^3+\sin\left(4x^2-2\right)\cdot(8x)\right]$

6. $y=\sqrt{\cos(5x+2)^3}$

$\dfrac{dy}{dx}=\frac{1}{2}\left[\cos(5x+2)^3\right]^{-\frac{1}{2}}\cdot\left(-\sin(5x+2)^3\right)\cdot\left(3(5x+2)^2\right)\cdot(5)$

7. $g(x)=\sin^3(\cos(2x))$

$g'(x)=3\sin^2(\cos(2x))\cdot(\cos(\cos(2x)))\cdot(-\sin(2x))\cdot(2)$

8. $h(x)=\sin\sqrt{x}+\sqrt{\sin x}$

$h'(x)=\cos\sqrt{x}\cdot\left(\frac{1}{2}x^{-\frac{1}{2}}\right)+\frac{1}{2}(\sin x)^{-\frac{1}{2}}\cdot(\cos x)$

9. Use the given table of values to find the following derivatives:

x	$f(x)$	$f'(x)$
2	1	7
8	5	-3

$g'(2)$ where $g(x)=[f(x)]^3$

$g'(x)=3[f(x)]^2\cdot f'(x)$

$g'(2)=3[1]^2\cdot 7=\boxed{21}$

$h'(2)$ where $h(x)=f(x^3)$

$h'(x)=f'(x^3)\cdot 3x^2$

$h'(2)=f'(8)\cdot 3(4)=(-3)(12)=\boxed{-36}$

10.

x	f(x)	$f'(x)$	g(x)	$g'(x)$
-1	2	3	2	-3
2	0	4	1	-5

Find $F'(-1)$ *where* $F(x)=f(g(x))$

$F'(x)=f'(g(x))\cdot g'(x)$

$F'(-1)=f'(g(-1))\cdot g'(-1)$

$=f'(2)\cdot(-3)$

$=4\cdot(-3)=\boxed{-12}$

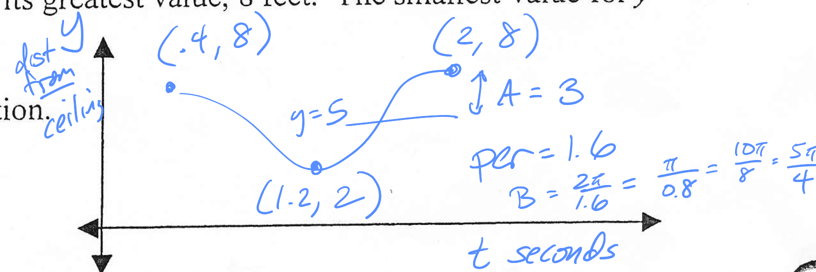
Find $G'(-1)$ *where* $G(x)=g(f(x))$

$G'(x)=g'(f(x))\cdot f'(x)$

$G'(-1)=g'(2)\cdot 3=(-5)(3)=\boxed{-15}$

11. A mass is bouncing up and down on a spring hanging from the ceiling. Its distance, y feet, from the ceiling varies sinusoidally with time t seconds, making a complete cycle every 1.6 seconds. At $t=.4$, y reaches its greatest value, 8 feet. The smallest value for y is 2 feet.

a) Draw a graph of the problem situation.



b) Write an equation for y in terms of t. $y=3\cos\left[\frac{5\pi}{4}(x-2)\right]+5$

c) How fast is the mass moving and in what direction *at* $t = 1?$ $t = 1.5?$ $t = 2.7?$

velocity $\longrightarrow$ $y'(t)=\frac{dy}{dt}=-3\sin\left[\frac{5\pi}{4}(x-2)\right]\cdot\left(\frac{5\pi}{4}\right)$

$y'(1)=-8.330$ $y'(1.5)=10.884$ $y'(2.7)=-4.508$

d) What is the fastest the mass moves?

use calc to find max of $y'(t)$

$t=1.6$ sec

$\boxed{y'(1.6)=\frac{15\pi}{4}\text{ feet/sec}}$