Ch 11 Chi-Square tests Handout

Name:

1. When is the Chi Square Goodness of Fit Test used?

When is the Chi-Square Test for Homogeneity used?

When is the Chi-Square Test for Independence (association) used?

2. Which example represents testing homogeneity and which represents testing independence?

## Let's see if you can tell the difference ....

Ex: A faculty of 36 professors is polled concerning their preference of the most recent negotiations. The results are presented in the following table.

Negotiations Preference						
Favor Do Not Favor Independen						
Male	11	7	2			
Female	7	8	1			

Negotiations Preference

Ex: A random sample of 20 males from the population of male professors in the school and another, independent, random sample of 16 females from the population of the female professors in the school.

Negotiations Preference					
	Favor	Do Not Favor	Inde		

	Favor	Do Not Favor	Independent
Male	11	7	2
Female	7	8	1

3.

Working out at 24-hour fitness is becoming more popular. Researchers interviewed 24-hour fitness members in two cities.

	City 1	City 2	Total
Men	38	68	
Women	203	150	

Is there a significant difference between the proportion of men and women customers in the 2 cities?

On the campus of a large boarding school, students are housed in Alpha, Beta, Gamma, and Delta dorms according to their grade levels (9,10,11,12). The schools officials have heard several complaints about the food services for the dorms, but felt that the complaints differed across the four dorms. A random sample of students was selected from each dorm and each student was asked "Are the current food services in your dormitory satisfactory?" Is there significant evidence to indicate whether the proportion of students satisfied with the current food services with different dorms?

	Alpha	Beta	Gamma	Delta	Total
Satisfactory	78	72	49	44	
Not Satisf.	22	28	26	31	
Total					

5. On January 1, 2004, it became mandatory for all police departments in Illinois to record data pertaining to race from every traffic stop. The town of Mundelein, IL collected data on at-fault drivers involved in car accidents in the village. The following is the distribution of race for users of roads in Mundelein.

White = 0.719, African American = 0.028, Hispanic = 0.207, Asian = 0.046

This next distribution represents the races of all 9868 drivers who were stopped for a moving violation in the village of Mundelein in a recent year.

White = 7079, African American =273, Hispanic = 2025, Asian = 491

a. Does the distribution of race in traffic stops indicate evidence of racial profiling at the 5% level of significance?

b. Compare observed and expected counts for each category. What does this information tell you?

## Quiz 11.1A AP Statistics Name:

1. A grocery store sells four different sizes of a popular brand of corn flakes. For the past few years the proportion of boxes they sell of each size has been quite stable: 10% Small, 15% Medium, 60% Large, and 15% Jumbo. They decide to change the pricing of the four sizes and want to see if this changes the proportion of boxes they sell of each size. To test this, a few weeks after changing the prices they take a simple random sample of 120 transactions involving corn flakes and count how many boxes of each size were sold. Here are the results.

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Small	Medium	Large	Jumbo
8	24	61	27

- (a) We wish to carry out a test of significance to see if the distribution of sizes of cereal boxes sold has changed. State the null and alternative hypotheses for this test.
- (b) Find the expected counts for each size box under the assumption that the null hypothesis is true.

#### Expected number of boxes sold for each box size

Small	Medium	Large	Jumbo	

(c) Discuss whether the conditions for this test have been met.

(d) Find the value of the test statistic and the *P*-value of the test, and make the appropriate conclusion. Use  $\alpha = 0.05$ .

(e) Based on your answer to (d), which error is it possible that you have made, Type I or Type II? Describe that error in the context of the problem.

(f) Use the components of the chi-square statistic to perform a follow-up analysis on the impact of the new prices on the sales of different sizes of cereal boxes.

### **Chapter 11 Solutions**

### Quiz 11.1A

**1.** (a)  $H_0$ : The distribution of sizes of all boxes sold of this brand of cereal did not change when the prices changed.  $H_a$ : The distribution of sizes of all boxes sold of this brand of cereal changed when the prices changed. (b) Expected counts: Small: (0.10)(120) = 12,

Medium: (0.15)(120) = 18, Large: (0.60)(120) = 72, Jumbo: (0.15)(120) = 18. (c) *Random*: the data come from a simple random sample of sales records. *10%*: We must assume that there were more than 1200 boxes of cereal sold. *Large counts*: All the expected counts (see part (b)) are at least 5.

(d) 
$$\chi^2 = \frac{(8-12)^2}{12} + \frac{(24-18)^2}{18} + \frac{(61-72)^2}{72} + \frac{(27-18)^2}{18} = 9.514; df = 3; P-value 0.023 (Using Table)$$

C, 0.02 < P-value < 0.025). Since the *P*-value is less than  $\alpha = 0.05$ , we can reject  $H_0$ . There is convincing evidence that the distribution of size of boxes sold changed after the prices changed. (e) It's possible that we have made a Type I error, which is concluding that the distribution of box sizes sold has changed when it has not. (f) Components of the chi-square statistic: Small: 1.33, Medium: 2.00, Large: 1.68, Jumbo: 4.5. The observed count of jumbo-sized boxes was much larger than the expected count, so it appears than an increase in sales of jumbo-sized boxes was the biggest impact of the price changes.

# Quiz 11.2AAP StatisticsName:

1. A radio station trying to determine what kind of music to play takes a simple random sample of 50 students at each of three locations: a local middle school, a high school, and a college. The students are asked to choose which of three different music genres they most enjoy hearing on the radio. Here are the results:

			Music Genre		
		Нір Нор	Alternative	Post-rock	Total
1 ~~	Middle School	28	18	4	50
Age	High School	22	22	6	50
College	College	16	20	14	50
	Total	66	60	24	150

(a) In the table below, provide the appropriate conditional distributions based on the data collected for comparing the music-listening preferences of the three age levels, based on the data above. Music Genre

			Whate Ochie	
		Нір Нор	Alternative	Post-rock
	Middle School			
Age level	High School			
	College			

(b) Make a graph that illustrates these conditional distributions effectively, and use the table from (a) and your graph to describe the relationship between age level and preferred music genre.

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(c) Perform the appropriate statistical test to determine if there is a difference in the music preference of these three age groups.

(d) If you chose a chi-square test for homogeneity in part (c), explain how the data could have been obtained to make a chi-square test for independence appropriate. If you chose a test for independence, explain how the data could have been obtained to make a test for homogeneity appropriate.

#### Quiz 11.2A

1. (a) & (b) Since age level is the explanatory variable, we calculate conditional distributions for each age level. See table and graph below (two options for graphs are shown: parallel and segmented bar graphs.) The data suggest that middle school students strongly prefer Hip Hop, most high school students are evenly divided between Hip Hop and Alternative, and college students are more evenly divided between all three genres. (c) State: We are testing the hypothesis  $H_{a}$ : The distribution of music genre is the same for the populations in all three age groups, against  $H_a$ : The distribution of music genre is not the same for the populations in all three age groups. We will use a significance level of  $\alpha = 0.05$ . Plan: The procedure is a chi-square test for homogeneity. Conditions: *Random*: the data come from SRSs of each population. 10%: The radio station's listening audience is likely to be more than 10 times the sample size. Large counts: All expected counts are at least 5 (see expected counts table below). Do: Using a calculator,  $\chi^2 = 10.673$ ; df = 4; P - value = 0.0305. (From Table C, *P*-value is between 0.025 and 0.05). Conclude: Since the *P*-value is smaller than  $\alpha = 0.05$ , we can reject  $H_0$ . There is convincing evidence that the distribution of music genre is different for the populations in the three age groups. (d) The test in (c) was a test of homogeneity. If we had treated all of the station's listeners as a single population from which we took one SRS, and if we treated age level and preferred music genre as two categorical variables, then a test of independence would have been appropriate.

Conditional	distributions	for	part (	a	)
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		Нір Нор	Alternative	Post-rock
Age level	Middle School	0.56	0.36	0.08
	High School	0.44	0.44	0.12
	College	0.32	0.40	0.28



Expected counts for part (c)

		Нір Нор	Alternative	Post-rock
Age level	Middle School	22	20	8
	High School	22	20	8
	College	22	20	8

#### Chi-Square Review

#### Provide a complete solution to each of the following problems.

- 1. Of 150 Democrats interviewed, 90 favor a proposition, and of 120 Republicans interviewed, 80 favor it. Is the true proportion of individuals in favor of the proportion significantly larger among Republicans than among Democrats at the 10% level of significance?
- 2. Does the color of a car influence the chance that it will be stolen? The Associate Press reported the following for a sample of 830 stolen vehicles: 140 were white, 100 were blue, 270 were red, 230 were black, and 90 were other colors. Suppose it is known that 15% of all cars are white, 15% are blue, 35% are red, 30% are black, and 5% are other colors. At the 0.01 level, does it appear that cars are stolen in proportion to the population color proportions?
- 3. Suppose, according to a survey conducted in 1960, the probability distribution of the age of an adult that is attending a 4 year university. In 1993, when a sample of 1000 adults attending a university was interviewed the following frequency distribution was observed. Is there significant evidence to conclude that the distribution of adults in a 4-year university is the same as that in 1960?

Age Group	Percent	Observed	
	Distribution	Values from	
	from 1960	1993	
17-18	25%	270	
19-20	23%	202	
21-22	20%	180	
23-24	15%	160	
25-26	8%	88	
27 and over	9%	100	

4. In the Lamorinda (Lafayette-Moraga-Orinda in the East Bay) community, 360 randomly picked people were classified according to their age group and political leaning. The data are as follows:

	Age Group			
Political	20-35	36-50	Over 50	
Leaning				
Conservative	30	25	35	
Moderate	80	85	45	
Liberal	40	10	10	

Is there significant evidence to conclude that a person's age and political leaning are not related?

5. The article "Should Dentists Advertise" compared the attitudes of consumers and dentists toward the advertising of dental services. Separate random samples of 101 consumers and 124 dentists were asked to respond to the statement, "I favor the use of advertising by dentists to attract new patients." The authors were interested in determining whether the two groups differed in their attitudes toward advertising. The data were as follows:

	Response				
	Strongly Agree	Agree	Neutral	Disagree	Strongly Disagree
Consumers	34	49	9	4	5
Dentists	9	18	23	28	46

Do the two groups seem to differ in their attitudes toward advertising?