Section 11.1 Chi-Square Test for Goodness of Fit (GOF)

- A **one-way table** is often used to display the distribution of a single categorical variable for a sample of individuals.
- The **chi-square test for goodness of fit** tests the null hypothesis that a categorical variable has a specified distribution in the population of interest.
- This test compares the **observed count** in each category with the counts that would be expected if H_0 were true. The **expected count** for any category is found by multiplying the sample size by the proportion in each category according to the null hypothesis.
- The **chi-square statistic** is

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

  where the sum is over all possible categories.
- The conditions for performing a chi-square test for goodness of fit are:
  - **Random:** The data were produced by a well-designed random sample or randomized experiment.
    - **10%:** When sampling without replacement, check that the population is at least 10 times as large as the sample.
  - **Large Counts:** All expected counts must be at least 5.
- When the conditions are met, the sampling distribution of the statistic χ^2 can be modeled by a **chi-square distribution**.
- Large values of χ^2 are evidence against H_0 and in favor of H_a. The P-value is the area to the right of χ^2 under the chi-square distribution with degrees of freedom df = number of categories − 1.
- If the test finds a statistically significant result, consider doing a follow-up analysis that compares the observed and expected counts and that looks for the largest **components** of the chi-square statistic.

Be prepared to state how observed compares with expected for the largest component of the Chi-Square test statistic. For example, “20 red M&Ms were observed compared with only 5 expected, which contributed to the largest component of the Chi-Square test statistic.”

Chi-Square Test for Goodness of Fit

1 Sample (One Way Table)

- **H_0:** The distribution of _____ is correct
- **H_a:** There distribution of_____ is incorrect

df = categories - 1

Conditions:

- Random
- 10%
- All expected counts ≥ 5

Calculator

- TI-84: χ^2 GOF Test
- TI-nspire: χ^2 GOF…

Sections 11.2 & 11.3

- **Chi-Square Test for Homogeneity** (comes from a Stratified Random Sample)
    - Check that the data come from two or more random samples OR two or more groups in a randomized experiment

- **Chi-Square Test for Independence** (comes from an Simple Random Sample [SRS] and then blocked)

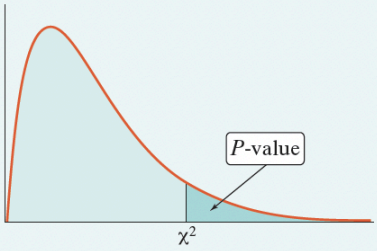
Summary of Chi-Square Tests

GOF: One variable in one population
Homogeneity: one variable in two or more populations (groups)
Independence: two variables in one population

- We can use a two-way table to summarize data involving two categorical variables. To analyze the data, we compare the conditional distributions of one variable for each value of the other variable. Then we turn to formal inference. Two different ways of producing data for two-way tables lead to two different types of chi-square tests.
- Some studies aim to compare the distribution of a single categorical variable for each of several populations or treatments. In such cases, researchers should take independent random samples from the populations of interest or use the groups in a randomized experiment. The null hypothesis is that there is no difference in the distribution of the categorical variable for each of the populations or treatments. We use the **chi-square test for homogeneity** to test this hypothesis.

- The conditions for performing a chi-square test for homogeneity are:
  - **Random:** The data come from independent random samples or the groups in a randomized experiment.
    - **10%:** When sampling without replacement, check that the population is at least 10 times as large as the sample.
  - **Large Counts:** All expected counts must be at least 5.
- Other studies are designed to investigate the relationship between two categorical variables. In such cases, researchers take a random sample from the population of interest and classify each individual based on the two categorical variables. The **chi-square test for independence** tests the null hypothesis that there is no association between the two categorical variables in the population of interest. Another way to state the null hypothesis is H_0: The two categorical variables are independent in the population of interest.
- The conditions for performing a chi-square test for independence are:
  - **Random:** The data come from a well-designed random sample or randomized experiment.
    - **10%:** When sampling without replacement, check that the population is at least 10 times as large as the sample.
  - **Large Counts:** All expected counts must be at least 5.
- The **expected count** in any cell of a two-way table when H_0 is true is

$$\text{expected count} = \frac{\text{row total} \cdot \text{column total}}{\text{table total}}$$



- The **chi-square statistic** is

$$\chi^2 = \sum \frac{(\text{Observed} - \text{Expected})^2}{\text{Expected}}$$

where the sum is over all cells in the two-way table.

- Both types of chi-square tests for two-way tables compare the value of the statistic χ^2 with critical values from the chi-square distribution with df = (number of rows − 1)(number of columns − 1). Large values of χ^2 are evidence against H_0 and in favor of H_a, so the *P*-value is the area under the chi-square density curve to the right of χ^2.
- If the test finds a statistically significant result, consider doing a follow-up analysis that compares the observed and expected counts and that looks for the largest components of the chi-square statistic.

As with ANY significance test, DO NOT calculate ANYTHING by hand. You are liable to make mistakes, and it is a waste of your time. Please spend your time writing a good set of hypotheses, providing ALL expected counts (from your calculator), drawing a nice graph, and writing a great conclusion. This is a recommendation from the College Board

The Chi-Square graph should start at zero, look skewed to the right, and be shaded above the Chi-Square statistic. Do not draw anything that looks like a Normal curve. There is NO NORMALITY with Chi-Square EVER.

Make sure to draw tick marks on the x-axis and the peak at df - 2.

Chi-Square Test for Homogeneity

Separate Samples (2 way table)

- H_0: There is no difference in **categorical variable** distribution for **population 1 and population 2**
- H_a: There is a difference in **categorical variable** distribution for **population 1 and population 2**

Chi-Square Test for Independence

One Sample (2 way table)

- H_0: There is not an association between___&___
- H_a: There is an association between ___&___

Same conditions for both

- Expected counts = (row total)(column total) / (table total)
- df = (rows - 1)(cols - 1)

Conditions:

- Random
- 10%
- Expected counts at ≥ 5 (col total)(row total)/(table total)

Calculator

- TI-84: χ^2 Test
- TI-nspire: χ^2 2-Way Test

Interpreting P-Values:

Assuming the null hypothesis is true, we have a ____ probability of getting a chi-square value of ____ or more extreme **due to chance.**