Key

Chapter 6 Review Sheet

1. A study of working couples measures the income X of the husband and the income Y of the wife in a large number of couples in which both partners are employed. Suppose that you knew the means μ_x, μ_y and the variances σ_x^2, σ_y^2 of both variables in the population.

a) Is it reasonable to take the mean of the total income to be $\mu_x + \mu_y$? Is it reasonable to take the variance of the total income to be $\sigma_x^2 + \sigma_y^2$?

Rule of Means: $\mu_{x+y} = \mu_x + \mu_y$

Yes, it is reasonable

Rule of Variances: $\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2$

2. Rotter Partners is planning a major investment. The amount of profit X is uncertain, but a probabilistic estimate gives the following distribution (in millions of dollars):

Profit	1	1.5	2	4	10
Probability	0.1	0.2	0.4	0.2	0.1

a) Find the mean profit μ_x, and the standard deviation of the profit.

$E(x) = 0.1(1) + 0.2(1.5) + 0.4(2) + 0.2(4) + 0.1(10) = \$3$ $\quad \sigma = \$2.52$

$\sigma^2 = 0.1(1-3)^2 + 0.2(1.5-3)^2 + 0.4(2-3)^2 + 0.2(4-3)^2 + 0.1(10-3)^2 = 6.35$

b) Rotter Partners owes its source of capital a fee of 200,000 plus 10% if the profits X. So the firm actually retains

Y= 0.9X-0.2 from the investment. Find the mean and standard deviation of Y.

$\mu_y = 0.9 \cdot \mu_x - 0.2 \quad \mu_y = 2.5$

$\sigma_y = 0.9\sigma_x$ since σ only changes with multiplication $\sigma_y = 2.268$

3. The time taken by Nick to do his statistics homework follows a *normal distribution* with mean 25 minutes and standard deviation 5 minutes. Jasmin's homework time also follows a normal distribution with mean 50 minutes and standard deviation 10 minutes. Nick's homework completion is independent of Jasmine's homework completion.

a) Define the random variable Z to be the difference between the amount of time spent on a randomly selected assignment of Jasmin's and a randomly selected assignment of Nick's. So Z = N – J. Find the mean and the standard deviation of Z.

$\mu_z = \mu_{n-j} = \mu_n - \mu_j = 25 - 50 = -25$

$\sigma_z = \sigma_{n-j} = \sqrt{\sigma_n^2 + \sigma_j^2} = \sqrt{25+100} = 11.180$

b) Calculate the probability that Nick spent longer doing his assignment than Jasmin did. Show your method clearly.

$P(z > 0) = \text{normalcdf}(0, \infty, -25, \sqrt{125}) = 0.01267$

4. A random variable X has a probability distribution as follows where *m* is a positive constant:

X	0	1	2	3
P(X)	2*m*	3*m*	13*m*	2*m*

Find the probability that P(*X*<2.0).

$2m + 3m + 13m + 2m = 1 \quad m = 0.05 \quad P(x<2.0) = P(0) + P(1)$

$P(x<2.0) = 2(0.05) + 3(0.05) = .25$

0 1 | $\boxed{2 \quad 3 \quad 4}$

1 - binomcdf$(4, \frac{1}{7}, 1)$

5. A medicine is known to produce side effects in 1 in 7 patients taking it. Suppose a doctor prescribes the medicine to 4 unrelated patients. What is the probability that ...

B - Either develops side effects or doesn't
I - Each patient is independent
T - 4 trials
S - Always $\frac{1}{7}$ chance of side effects

   a. none of the patients will develop side effects.

   $P(X=0) = \binom{4}{0}\left(\frac{6}{7}\right)^4\left(\frac{1}{7}\right)^0 = .53978$

   b. at least two of the patients will develop side effects.

   $P(X \geq 2) = 1 - \text{binomcdf}(4, \frac{1}{7}, 1) = .1003$

   c. between 2 and 4 patients will develop side effects (find the exact prob and the normal approximation)

   $\mu_X = np$ Exact Probability: .1003 (Part b)

   $\sigma_X = \sqrt{np(1-p)}$ Normal Probability: normalcdf$\left(2, 4, \frac{1}{7}\cdot 4, \sqrt{\frac{1}{7}\cdot 4 \cdot \frac{6}{7}}\right) = \boxed{.02061}$

   Not similar because normal approx. conditions are not met!

~~6.~~ The average number of calories in a Clark candy bar are Normally distributed in a candy bar with mean 200 and standard deviation 15 calories. 35 Clark Bars are selected at random. What is the probability that the average number of calories...

   a. exceeds 205 calories?

   b. falls between 192 and 207 calories?

SKIP

7. What is the probability that you first roll a "3" on the roll of a fair die on the 5th roll?

$\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) = 0.08038$ B - Either 3 or not 3 I - Each roll is independent N - 5 rolls S - Always $\frac{1}{6}$ chance

8. An experiment consists of rolling a die until a prime number (2, 3, or 5) is observed. Let X = number of rolls required to get the first prime number.

   a. Verify that X has a geometric distribution.

   B - Either Prime or Not Prime I - Each roll is independent

   N - X trials to 1st prime S - $\frac{1}{2}$ chance of Prime each time

   b. Construct a probability distribution table to include at least 5 entries for the probabilities of X. Record the probabilities to four decimal places.

   $P(X=1) = \left(\frac{1}{2}\right)$ $P(X=2) = \left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right) = \frac{1}{4}$

   $P(X=3) = \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$ $P(X=4) = \left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)$

   $P(X=5) = \left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)$

X	1	2	3	4	5
P(X)	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	$\frac{1}{32}$

   c. What is the probability that the first time you roll a prime number is on the 8th roll.

   $P(X=8) = \left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right) = .003906$

9. What is the probability that it takes you less than 5 rolls to roll your first "3"?

$P(X<5) = \text{geometcdf}(\frac{1}{6}, 4) = .5177$

10. At a local car dealership, 30% of the cars are manual. What is the probability that ...

B - Either Manual or Not
I - Each car is independent
N - 5 trials
S - $\frac{3}{10}$ chance of manual

   a. the 5th car sampled is the first to be a manual?

   $P(X=5) = \left(\frac{7}{10}\right)^4\left(\frac{3}{10}\right) = .07203$

   b. you must sample at least four cars before finding one that is a manual?

   $P(X>4) = 1 - \text{geometcdf}(\frac{3}{10}, 4) = .2401$

11. Over the holiday's, *Amazon.com* shipped many books to various locations. The mean weight of a book shipped was 5 pounds with a standard deviation of 2.5 pounds. The mean weight of the packaging material was 0.6 pounds with a standard deviation of 0.3. Keep in mind that a **box** consists of the book and also the packaging material. Assume Normally distributed.

   a. What is the mean weight of a **box**?

   $\mu_{box} = \mu_{Book \,\&\, Packaging} = \mu_{Book} + \mu_{packaging} = 5 + 0.6$ $\boxed{\mu_{box} = 5.6 \text{ pounds}}$

   b. What is the standard deviation of the mean weight of a box?

   $\sigma_{box} = \sigma_{Book \,\&\, Packaging} = \sqrt{\sigma^2_{book} + \sigma^2_{packaging}} = \sqrt{2.5^2 + .3^2}$ $\boxed{\sigma_{box} = 2.518 \text{ pounds}}$

   c. What is the probability that the mean weight of a box is at most 7 pounds?

   $P(X \leq 7) = \text{normalcdf}(0, 7, 5.6, 2.518) = .6978$