KEY **Chapter 8 Review Sheet** KEY

1. The systolic blood pressure of a random sample of 100 female students has sample mean $\bar{x} = 116.80$ and a standard deviation of 20. Give a 90% confidence interval for the mean blood pressure in the population.

1 mean t-interval or $\bar{x} \pm t^*\left(\frac{S_x}{\sqrt{n}}\right)$

SRS - Stated
10% $100(10) \le N$
≈Normal: CLT $100 \ge 30$

using calculator:
tinterval
$\bar{x} = 116.80, S_x = 20, n = 100$
$(113.479, 120.121)$ df = 99

I am 90% confident that the true mean blood pressure for female students lies between 113.52 and 120.08

2. Suppose the directions in #1 told you to give a 95% confidence interval. Would this *increase* or *decrease* the size of the confidence interval? Explain.

As the confidence level increases, so does the critical value, t^*. When t^* increases, the margin of error increases, so the size of the C.I. will also increase.

3. Suppose you were told that only 50 students were available for question #1. Would this *increase* or *decrease* the width of the confidence interval? Explain.

Since n is a value in the denominator of the margin of error, as n decreases, the margin of error increases, so the width of the C.I. will also increase.

4. For question #1, what does a 90% confidence level mean (in context)?

When many samples of the same size are taken, we expect that 90% of the resulting confidence intervals will capture the true mean blood pressure for female students.

5. A scale used in analytical chemistry gives weights in repeated weighings of the same object that have a normal distribution with mean equal to the true weight of the object (that is, the scale is unbiased). The standard deviation of repeated weighing of the same object is $\sigma = 0.01$. The student wants to estimate the true weight within ± 0.01 gram with 99% confidence. How many weighings should she average to achieve this? Since we do not have a sample yet, and do not know n, then using margin of error $= t^* \frac{S_x}{\sqrt{n}}$ is not useful because t^* is based on knowing n. Therefore, use margin of error $= z^* \frac{\sigma}{\sqrt{n}} = 2.576\left(\frac{0.01}{\sqrt{n}}\right) = .01$
Solve for n to get 6.635, then round up to $\boxed{7}$.

6. The distribution of scores on a Scholastic Aptitude Test among Indiana University-bound high school seniors is normal. The standard deviations of scores from samples taken previously have been close to 90. This value is quite stable from year to year, but the mean changes a bit. How large a sample would be required to estimate the population mean within ± 10 points with 95% confidence?

use same logic as in problem #5

$ME = z^* \cdot \frac{\sigma}{\sqrt{n}} = 1.96\left(\frac{90}{\sqrt{n}}\right) = 10$

Solve for $n = 311.17$, Round up to $\boxed{312}$.

7. What are the conditions for a 1 proportion z-interval?

SRS? 10% condition $10n \le N$? Normality $n\hat{p} \ge 10$, $n(1-\hat{p}) \ge 10$?

8. What is the margin of error for a 1-proportion z-interval?

$$ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

9. A survey is to be conducted to determine the proportion of US voters who support additional aid to African nations to fight poverty and HIV/AIDS. Of the proposed sample sizes, which is the smallest that will guarantee a margin of error of no more than 4% for a 90% confidence level?

a. 225 b. 325 c. 423 d. 525 e. 625

$ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ since we do not know $\hat{p}$, use $p^* = .5$

$ME = 1.645 \sqrt{\frac{.5(.5)}{n}}$

Solve for $n = 422.89$, Round up to 423

10. A sample of 45 Turkey De-Lite hot dogs yielded a mean sodium content of 290 mg with a standard deviation of 28 mg. Construct a 98% confidence level for the average sodium content in Turkey De-Lite hot dogs.

a) $290 \pm 1.96(4.174)$ b) $290 \pm 2.02(4.174)$ c) $290 \pm 2.33(4.174)$
d) $290 \pm 2.41(4.174)$ e) $290 \pm 2.69(4.174)$

$$290 \pm t^*\left(\frac{28}{\sqrt{45}}\right) \rightarrow 290 \pm 2.414\left(\frac{28}{\sqrt{45}}\right)$$

11. The bookstore of a large university wants to determine how much money full-time first-year students spend, on average, in the university bookstore. At the end of school year a random sample of bookstore bills for 40 first-year students is obtained. A 95% confidence interval for the mean is calculated to be ($450, $1300). Which of the following is a correct interpretation of this *interval*?

a. We can be 95% confident that the average bookstore bill for full-time first-year students in our sample is between $450 and $1300. NO! For population.

b. We can be 95% confident that the average bookstore bill for full-time first-year students is between $450 and $1300.
tricky. The population of the problem is full-time first-year students only at this particular university!

c. We can be 95% confident that the average bookstore bill for full-time first-year students at this university is between $450 and $1300.
yes!

d. The average bookstore bill for full-time first-year students at this university will be between $450 and $1300 about 95% of the time. NO.

e. About 95 out of 100 full-time first-year students at this university will have bookstore bills between $450 and $1300. NO.

12. A manufacturer constructs a 95% confidence interval for the average weight of the items he manufactures. His results need to be included in a report to his superiors, and the resulting interval is wider than he would like. In order to decrease the size of the interval the *most*, the manufacturer should take a new sample and.

a. increase the confidence level and increase the sample size.
b. decrease the confidence level and increase the sample size.
c. increase the confidence level and decrease the sample size.
d. decrease the confidence level and decrease the sample size.
e. The manufacturer will not be able to decrease the size of the interval.

$ME = t^* \cdot \frac{S_x}{\sqrt{n}}$

To decrease ME, decrease t^* and increase n.
By decrease t^*, we are also decreasing the confidence level.

13. Owners of a day-care chain wish to determine the proportion of families in need of day care for the town of Bockville. Bockville is estimated to have 100 families. The owners of the day-care chain randomly sample 50 families and find that 60% of them have a need for day-care services. Which of the following is a condition necessary for constructing a confidence interval for a proportion that has not been met?

$N=100, n=50, \hat{p}=.60$

a. The data constitute a representative random sample from the population of interest.
Stated

b. The sample size is less than 10% of the population size.
10% condition: $50(10) \leq N$? NO!

c. The counts of those who need day care and those who don't care are 10 or more.
$np \geq 10,\ n(1-p) \geq 10 \rightarrow 50(.6) \geq 10,\ 50(.4) \geq 10$ yes

d. The distribution of sample values is approximately normally distributed.
Yes, because of Large Counts in part c.

e. All conditions necessary for constructing a confidence interval for the proportion seem to be met.
No, see part b.

14. In a news article on SFgate.com (SF chronicle online) last year, addressing the issue of Mayor Newsom seeking rehab for his alcohol abuse. Some voters believe that his attempt at counseling is really to save his political career. In a recent survey, 56 out of 85 people agreed that Mayor Newsom's statement to attend counseling for his alcohol abuse problem is a political move. Find a 90% confidence interval for the true proportion of voters that believe that Mayor Newsom attempt at counseling is politically motivated.

1 proportion z-interval

SRS assume

10% $85(10) \le N$

$\approx$ normal $85\left(\frac{56}{85}\right) \ge 10$

$85\left(\frac{29}{85}\right) \ge 10$

use calculator

(.574, .743)

$X = 56,\ n = 85,\ C = 90$

$\hat{p} = .6588$

$ME = .0845$

I am 90% confident that the true proportion of voters who believe Newsom's counseling is politically motivated is between .574 and .743.

15. The manufacturer's of Vicious Victuals (VV) dog food want to know the proportion of dogs that prefer their product to other brands. They randomly select a sample of 589 dogs, and present each dog with a bowl of VV and a bowl of another brand of dog food. Of this sample, 374 dogs chose VV.

a. *Can we estimate the population parameter using the normal approximation to the binomial in this scenario? Explain*

$np \ge 10?$ $589\left(\frac{374}{589}\right) \ge 10$ yes

$n(1-p) \ge 10?$ $589\left(1 - \frac{374}{589}\right) \ge 10$ yes

b. What's the estimate of the proportion?

$\hat{p} = \frac{374}{589} = .635$

c. What critical z-value would you use to conduct a 95% confidence interval?

$z^* = \text{invnorm}(.975, 0, 1) = 1.96$

d. In general, what does the confidence interval give you (in context)?

The confidence interval gives us a range of values that will capture the true proportion of dogs that prefer VV dog food.

e. Calculate the margin of error for the 95% confidence interval for the population proportion.

$$ME = z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96\sqrt{\frac{(.635)(.365)}{589}} = .0388$$