C9D2

One Proportion Z-Test Practice

Example #1: ***Better to be last?***

On shows like *American Idol,* contestants often wonder if there is an advantage to performing last. To investigate, earchers selected a random sample of 600 college students and showed each student the audition nce that the true proportion of students who prefer the final singer is greater than p = 1/12. For each student, the videos were shown in random order. So we would expect approximately 1/12 of the students to prefer the last singer they view, assuming the order doesn't matter. In this study, 59 of the 600 students preferred the last singer they viewed. ***Do these data provide convincing evidence at the 5% significance level that there is an advantage to going last?***

Step 1: We want to test the following hypotheses at the α = 0.05 significance level:

$$H_0 : p = 1/12$$

$$H_a : p > 1/12$$

where p = the true proportion of students who prefer the last singer they see.

Step 2: If conditions are met, we will perform a one-sample z test for p.

- Random: A random sample of students was selected, and the order in which the videos were viewed was randomized for each subject.
  - 10%: It is reasonable to assume that there are more than 10(600) = 6000 students.
- Large Counts: np_0 = 600(1/12) = 50 $\geq$ 10, $n(1-p_0)$ = 600(1 – 1/12) = 550 $\geq$ 10.

Step 3: The sample proportion of fans who preferred the last contestant is $\hat{p}$ = 59/600 = 0.098.

- **Test statistic** $z = \dfrac{0.098-0.083}{\sqrt{\dfrac{0.083(1-0.083)}{600}}} = 1.33$
- ***P*-value** $P(Z > 1.33) = 1 - 0.9082 = 0.0918$.

  Using technology: The command normalcdf(lower:1.33, upper:1000, μ :0, σ :1) gives a *P*-value of 0.0918.

Step 4 Conclude: Because the *P*-value of 0.0918 is greater than α = 0.05, we fail to reject the null hypothesis. There is not convincing evidedence that there is an advantage to performing last.

Example #2: ***Benford's law and fraud***

When the accounting firm AJL and Associates audits a company's financial records for fraud, they often use a test based on Benford's law. Benford's law states that the distribution of first digits in many real-life sources of data is not uniform. In fact, when there is no fraud, about 30.1% of the numbers in financial records begin with the digit 1. If the proportion of first digits that are 1 is significantly different from 0.301 in a random sample of records, AJL and Associates does a much more thorough investigation of the company. Suppose that a random sample of 300 expenses from a company's financial records results in only 68 expenses that begin with the digit 1. Should AJL and Associates do a more thorough investigation of this company? Justify your answer.

Step 1: We want to test the following hypotheses at the α = 0.05 significance level:

$$H_0 : p = 0.301$$

$$H_a : p \neq 0.301$$

where p = the proportion of all expenses that begin with the digit 1 for this company.

Step 2: If conditions are met, we will perform a one-sample z test for p.

- *Random:* A random sample of expenses was selected.
- 10%: It is reasonable to assume that there are more than 10(300) = 3000 expenses in this company's financial records.
- *Large Counts:* np_0 = 300(0.301) = 90.3 $\geq$ 10, $n(1-p_0)$ = 300(1 – 0.301) = 209.7 $\geq$ 10.

Step 3: The sample proportion of expenses that began with the digit 1 is $\hat{p}$ = 68/300 = 0.227.

- **Test statistic** $z = \dfrac{0.227-0.301}{\sqrt{\dfrac{0.301(1-0.301)}{300}}} = -2.79$
- ***P*-value** $2P(Z < -2.79) = 2(0.0026) = 0.0052$. *Using technology*: The calculator's 1-PropZTest gives $z = -2.807$ and *P*-value = 0.0050.

Step 4 -Conclude: Because the *P*-value of 0.0050 is less than $\alpha = 0.05$, we reject the null hypothesis. There is convincing evidence that the proportion of all expenses that have a first digit of 1 for this company is not 0.301. Therefore, AJL and Associates should do a more thorough investigation of this company.

Example #3: ***Benford's law and fraud***

Problem:

(a) Find and interpret a confidence interval for the proportion of all expenses that begin with the digit 1 for the company in the previous Alternate Example.

(b) Does your interval from part (a) lead to the same conclusion as the significance test? Explain.

Solution:

(a) **State:** We want to estimate p = the proportion of all expenses that begin with the digit 1 for this company at the 95% confidence level.

Plan: We will use a one-sample z interval for p if the following conditions are satisfied.

- *Random:* A random sample of expenses was selected.
  - 10%: It is reasonable to assume that there are more than 10(300) = 3000 expenses in this company's financial records.
- *Large Counts:* $n\hat{p} = 68 \geq 10$ and $n(1-\hat{p}) = 232 \geq 10$

Do: $\hat{p} \pm z^*\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} = 0.227 \pm 1.96\sqrt{\dfrac{0.227(1-0.227)}{300}} = 0.227 \pm 0.047 = (0.180, 0.274)$

Conclude: We are 95% confident that the interval from 0.180 to 0.274 captures the proportion of all expenses at this company that begin with the digit 1.

(b) Yes. Because 0.301 is not in the interval, it is not a plausible value for the true proportion of expenses that begin with the digit 1. Thus, this company should be investigated for fraud.

Example #4: ***Spinning Heads?***

When a fair coin is flipped, we all know that the probability the coin lands on "heads" is 0.50. However, what if a coin is spun? According to the article "Euro coin accused of unfair flipping" in the *New Scientist* (January 4, 2002), two Polish math professors and their students spun a Belgian euro coin 250 times. It landed "heads" 140 times. One of the professors concluded that the coin was minted asymmetrically. A representative from the Belgian mint said the result was just chance.

Problem:

(a) State the hypotheses we are interested in testing.

(b) Assuming the conditions for inference are met, what values of $\hat{p}$ would lead to a rejection of H_0 at the 5% significance level? Explain how you obtained your answer.

(c) Based on your answer to part (b) and the results of the study, what conclusion would you draw?

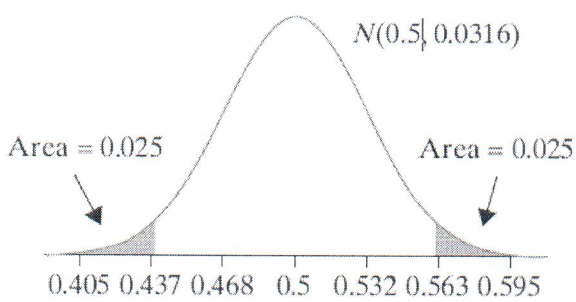
(d) Suppose that the actual probability of heads when spinning a Belgian euro coin is $p = 0.55$. What is the probability that a Type II error will be committed when using the results of 250 spins? That is, what is the probability that $\hat{p}$ does not fall in the rejection region from part (c) when p really equals 0.55?

(e) Based on your answer to part (d), what is the probability that you find convincing evidence that $p \neq 0.5$ when spinning a coin 250 times, assuming that p really equals 0.55?

lution:

(a) $H_0: p = 0.5$ versus $H_a: p \neq 0.5$ where p = the long-run proportion of heads when spinning a Belgian euro coin.

(b) The sampling distribution of $\hat{p}$ is approximately Normal with $\mu_{\hat{p}} = 0.5$ and $\sigma_{\hat{p}} = \sqrt{\frac{0.5(1-0.5)}{250}} = 0.0316$. Because the alternative hypothesis is two-sided, any values in the outer 2.5% of either tail of the sampling distribution of $\hat{p}$ would lead to a rejection of H_0 when $\alpha = 0.05$, as shown in the diagram below.



Sampling distribution of $\hat{p}$

Using Table A, the boundary values are at $z = -1.96$ and $z = 1.96$. Solving $\frac{\hat{p}-0.5}{0.0316} < -1.96$ and $1.96 < \frac{\hat{p}-0.5}{0.0316}$ indicates we should reject H_0 if $\hat{p} < 0.438$ or $\hat{p} > 0.562$.

(c) Because $\hat{p} = 140/250 = 0.56$ is not in either rejection region, we fail to reject H_0. These data do not provide convincing evidence that the long-run proportion of heads is different than 0.5 when spinning this Belgian euro coin.

(d) If $p = 0.55$, then $\mu_{\hat{p}} = 0.55$ and $\sigma_{\hat{p}} = \sqrt{\frac{0.55(1-0.55)}{250}} = 0.0315$. Thus, P(Type II error) = $P(0.438 < \hat{p} < 0.562)$ = $P\left(\frac{0.438-0.55}{0.0315} < Z < \frac{0.562-0.55}{0.0315}\right) = P(-3.56 < Z < 0.38) = 0.6480 - 0.0002 = 0.6478$. Assuming that p = 0.55, there is a 0.6478 probability that we do not find convincing evidence that the long-run proportion of heads differs from 0.5.

(e) If $p = 0.55$, there is a $1 - 0.6478 = 0.3522$ probability that we find convincing evidence that the long-run proportion of heads differs from 0.50.