BC Calculus – Hahn / Limburg
Chapter 10A Quiz 2024-25
No Calculators

Anderson Tanimoi is ready to get series about learning
Period: 4

	1	2	3	4-6	7	Totals (not an average)
Implementing Mathematical Practices	4	4	4	4		4
Connecting Representations	4					4
Justification	4		4		4	4
Communication and Notation	4		2	4		3

For every justification problem on this quiz, indicate the name of the convergence/divergence test used.

1. The alternating series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+2}$ converges to S.

a) If the sum of the first six terms of the series is used to approximate S, what is the alternating series error bound?

$|a_7| = \frac{(-1)^8}{14+2} = \boxed{\frac{1}{16}}$

b) Determine whether the alternating series is absolutely or conditionally convergent. Justify your answer.

Strictly alternating, $\lim_{n\to\infty} \frac{1}{2n+2} = 0$, $\frac{1}{2n+2} \geq \frac{1}{2(n+1)+2}$

true true true greater denominator

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By alternating series test, convergent.

must diverge

$\sum_{n=1}^{\infty} \frac{1}{2n+2}$ $\lim_{n\to\infty} \left| \frac{\frac{1}{2n+2}}{\frac{1}{n}} \right| = \lim_{n\to\infty} \left| \frac{n}{2n+2} \right| = \frac{1}{2}$, both same behavior, because $\frac{1}{n}$ diverges due to harmonic, $\frac{1}{2n+2}$ $\boxed{\therefore \text{conditional convergence}}$

Test used: alternating series + lim. comp. test

2. What are all the values for x for which $\sum_{n=0}^{\infty} \left(\frac{x}{2+x}\right)^n$ converges?

root test converges when result < 1

$\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{x}{2+x}\right)^n\right|} = \lim_{n\to\infty} \left|\frac{x}{2+x}\right|$

$\left|\frac{x}{2+x}\right| < 1$ • for all positive #s x $\frac{\text{smaller}}{\text{larger}} < 1$

• for $x=0$ $0<1$

so $\boxed{x > -1}$

• not for -1 $1 \not< 1$

• not everything less than -1 because |numerator| > |denom| → >1

3. Consider the series $\sum_{n=2}^{\infty} \frac{1}{n^p \ln(n)}$ where $p \geq 0$.

a) Show that the series **converges** for $p > 1$. **Identify the name of the test used.**

$$\lim_{n\to\infty} \left| \frac{\frac{1}{n^p \ln(n)}}{\frac{1}{n^p}} \right| = \lim_{n\to\infty} \left| \frac{1}{\ln(n)} \right| = 0 \qquad \frac{1}{n^p \ln(n)} << \frac{1}{n^p}$$

because $\frac{1}{n^p}$ converges when $p>1$ by p series test and $\frac{1}{n^p \ln(n)}$ is much less than $\frac{1}{n^p}$ which converges, $\sum \frac{1}{n^p \ln(n)}$ must $\boxed{\text{converge}}$ as well by limit comparison and comparison test.

Test used: P series, Lim. Comp., Comp.

denom always gets bigger

b) Use the **Integral Test** to determine whether the series converges or diverges for $p = 1$.

$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ always positive, decrasing, and continuous so we can use $\int$ test

$$\lim_{t\to\infty} \int_2^{\cancel{t}} \frac{1}{n\ln(n)}\,dn = \Big[\ln(\ln(n))\Big]_2^{\infty} = \infty - \ln(\ln(2))$$

diverg.

$\boxed{\text{divergent}}$

4. Use the **Root Test** to determine whether the series converges or diverges.

$\sum_{k=1}^{\infty} (-1)^k \frac{k^k}{e^{2k}}$

$\lim_{k\to\infty} \sqrt[k]{\left|(-1)^k \frac{k^k}{e^{2k}}\right|} = \lim_{k\to\infty} \left(\frac{k}{e^2}\right) = \infty$

$\infty > 1$ therefore $\boxed{\text{diverges}}$ by root test

5. Use the **Limit Comparison Test** to determine whether the series converges or diverges.

$\sum_{n=1}^{\infty} \frac{n+1}{n(n+2)(n+3)}$

$\lim_{n\to\infty} \left| \frac{\frac{n+1}{n(n+2)(n+3)}}{\frac{1}{n^2}} \right| = \lim_{n\to\infty} \left| \frac{n^3+n^2}{n^3+5n^2+6n} \right| = 1$

So both have same behavior. $\frac{1}{n^2}$ converges by p series test $2>1$, so by lim. comp test, $\frac{n+1}{n(n+2)(n+3)}$ must $\boxed{\text{converge}}$

6. For what values of p will BOTH of the following series converge?

$\sum_{n=1}^{\infty} \frac{1}{n^{(2p)}}$ and $\sum_{n=1}^{\infty} \left(\frac{p}{2}\right)^n$

p series: geom:

$2p > 1$

$\frac{p}{2} < 1$

$p < 2$

$p > \frac{1}{2}$

$> \boxed{\frac{1}{2} < p < 2}$

7. Determine whether each series converges or diverges. Justify your answer for each.

a) $\sum_{n=1}^{\infty} \frac{8^n}{n!}$

$\lim_{n\to\infty} \left| \frac{\frac{8^{n+1}}{(n+1)!}}{\frac{8^n}{n!}} \right| = \lim_{n\to\infty} \left| \frac{8}{n+1} \right| = 0$

$0 \le 0 < 1$ therefore converges by ratio test

Converge or Diverge? Converges

Test used: ratio test

b) $\sum_{n=1}^{\infty} \frac{n!}{n^{100}}$

$\lim_{n\to\infty} \left| \frac{\frac{(n+1)!}{(n+1)^{100}}}{\frac{n!}{n^{100}}} \right| = \lim_{n\to\infty} \left| \frac{(n+1)n^{100}}{(n+1)^{100}} \right|$

as $n \to \infty$, $(n+1)^{100} = n^{100}$

$\therefore \lim_{n\to\infty} \left| \frac{(n+1)n^{100}}{(n+1)^{100}} \right| = \infty$

$\infty > 1$, diverges by lim. comp.

Converge or Diverge? diverges

Test used: lim. comp.

c) $\sum_{n=1}^{\infty} \frac{2n}{n+3}$

$\lim_{n\to\infty} \left(\frac{2n}{n+3} \right) = 2$, by

n^{th} term test because

$2 \neq 0$, series diverges

Converge or Diverge? diverges

Test used: n^{th} term

d) $\sum_{n=1}^{\infty} \frac{-8}{(-3)^n}$

$-8 \sum_{n=1}^{\infty} \frac{1}{(-3)^n}$, $\lim_{n\to\infty} \sqrt[n]{\left| \frac{1}{(-3)^n} \right|} = \frac{1}{3}$

$\frac{1}{3} < 1$ so convergent

by root test

Converge or Diverge? Converge

Test used: root test