3. Consider the Taylor series for $f(x) = \frac{1}{1-\frac{x^2}{4}}$ centered at x = 0. $\frac{1}{1-y} = \sum x^n$, $\frac{1}{1-(\frac{x^2}{4})} = \sum \left(\frac{x^2}{4}\right)^n$

a) [CR] Write the first four terms of the series.

$$1 + \frac{x^2}{4} + \frac{x^4}{16} + \frac{x^6}{64}$$ ✓

b) [J, CN] Determine the interval of convergence for this series. Clearly show all your work (and justify with tests) to earn full credit.

root test $\sqrt[n]{\left|\left(\frac{x^2}{4}\right)^n\right|} < 1$, "convergent" $\left|\frac{x^2}{4}\right| < 1$, $-1 < \frac{x^2}{4} < 1$

$-4 < x^2 < 4$, $-2 < x < 2$ ✓

$x=2$: $\sum 1^n = \sum 1$, divergent by n^{th} term test

$x=-2$: $\sum 1^n = \sum 1$, divergent by n^{th} term test

$\boxed{-2 < x < 2}$

C) [CR] What is the radius of convergence for this series?

2 ✓

4. [IMP] Find a 2nd degree Taylor polynomial for $f(x) = \sqrt{x+1}$ centered at x = 3.

$T(x) = \sum \frac{f^{(n)}(a)(x-a)^n}{n!}$, $a=3$ $f(x)=\sqrt{x+1}$ $f'(x) = \frac{1}{2}(x+1)^{-\frac{1}{2}}$ $f''(x) = -\frac{1}{4}(x+1)^{-\frac{3}{2}}$

$f(a) = 2$ $f'(a) = \frac{1}{4}$ $f''(a) = -\frac{1}{32}$

$T(x) = 2 + \frac{1}{4}(x-3) - \frac{1}{32}\left(\frac{1}{2!}\right)(x-3)^2$

$T(x) = 2 + \frac{(x-3)}{4} - \frac{(x-3)^2}{64}$ ✓

Centered at 2 ✓

5. [IMP, J] Find the Lagrange error bound for the approximation of $\sqrt{e}$ using a 3rd degree Maclaurin polynomial of $f(x) = e^x$. $e^x = \sum \frac{x^n}{n!}$ $R \leq \left|\frac{\max[f^{n+1}(z)](x-a)^{n+1}}{(n+1)!}\right|$

$3^{1/2}$

$x = \frac{1}{2}, a = 1, n = 3$ $R \leq \left|\frac{(e^{\frac{1}{2}})(\frac{1}{2}-1)^4}{4!}\right|$, $R \leq \left|\frac{-e^{\frac{1}{2}}}{4! \, 2^4}\right|$

$4 \cdot 3 \cdot 2 = 24$

$2 \cdot 2 \cdot 2 \cdot 2 = 16$

$24 \cdot 16 = 384$

$R \leq \frac{\sqrt{3}}{384}$ ✓

BC Calculus 2024-25 - Hahn/Limburg
No Calculators

What did Anderson T. ask Mr. Series at the dining table?
Could you please push Ma-Chair In? Period: 4

Scoring Grid (teacher use only):

	1	2	3	4	5	Totals (not an average)
Implementing Mathematical Processes	4			4	4	4
Connecting Representations	4		4			4
Justification		4	4		4	4
Communication and Notation		4	4			4

1. Consider the Maclaurin series for $f(x) = \cos x$ $\sum \frac{(-1)^n x^{2n}}{(2n)!}$

a) [IMP] Use the first two terms of the Maclaurin series to approximate $\cos\left(\frac{1}{2}\right)$

$\left(1 - \frac{x^2}{2!}\right)\Big|_{x=1/2}$, $1 - \frac{1}{4(2)} = \frac{7}{8}$

b) [CR] Find the Alternating Series error bound for your answer from part (a).

$1 - \frac{x^2}{2!} + \frac{x^4}{4!}$ $\quad \frac{x^4}{4!}\Big|_{x=\frac{1}{2}} = \frac{1}{16 \cdot 4!}$ $\quad 4! = 4 \cdot 3 \cdot 2 = 24$

$\frac{1}{16 \cdot 24} = \frac{1}{384}$

$$\begin{array}{r} 24 \\ \times 16 \\ \hline 144 \\ +240 \\ \hline 384 \end{array}$$

2. [J, CN] Find the interval of convergence for $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n^2}$. Clearly show all your work (and justify with tests) to earn full credit.

$\lim_{n \to \infty} \left| \frac{S_{n+1}}{S_n} \right| < 1$, converges by ratio test

$\lim_{n \to \infty} \left| \frac{\frac{(x-3)^{n+1}}{(n+1)^2}}{\frac{(x-3)^n}{n^2}} \right| = \lim_{n \to \infty} \left| \frac{(x-3)n^2}{(n+1)^2} \right| < 1$, $|x-3| < 1$, $-1 < x-3 < 1$

$2 \le x \le 4$

$x=2$: $\frac{(-1)^n}{n^2}$, alt, always decreasing in abs, goes to zero, convergent

$x=4$: $\frac{1}{n^2}$, convergent by p-series test